A senior mathematics capstone course

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This semester I am teaching Mathematics 4513, the Mathematics Senior Seminar, for the second time.

It is a course designed to fulfill our University's Senior Capstone Experience requirement, which describes it as follows:

> "Designed to culminate a student's undergraduate field of study and place it in a larger social, intellectual and professional context, the capstone experience should be an intensive experience in the major of interdisciplinary field at the senior level of performance. The capstone must include an in-depth writing component."

Historically, our department has maintained no consistent content or approach to the course. Instructors have usually given either:

1. A fairly conventional course on some unusual topic. This is the approach I tried the first time I taught the course (with 26 students, the class was too large for a lot of individual work). The students gave the course good evaluations, but I felt dissatisfied.

2. A course built around individual presentations, or more often group presentations, on self-selected topics.

In our department (as opposed to some others), the course is generally regarded as an easy A or B.

This semester, the second time around and with the class limited to 10 students, I resolved to experiment with a much different format. My objectives include:

1. Fill some of the gaps that inevitably develop as students move through a formal curriculum.

2. Develop students' skills in communicating mathematics, both in writing and verbally, in formal and informal environments.

3. Increase the students' familiarity with the "culture" of mathematics.

In this talk, I'll report on the structure of the course, and how my efforts for each of the objectives seem to be going so far.

Our class meets in a seminar room in the Main Library. It has a long table with five students on each side, and a small blackboard at one end. It is more suited to discussion than to lecturing, but is workable for either.

Objective # 1: Fill in "gaps".

I have given lecture series on several topics:

1. "Methods of proof" (4 days)

2. "The art of writing mathematics and presenting talks" (2 days)

3. "Crash course in linear algebra" (5 days)

4. "Intro to TEX and LATEX" (2 days)

Additional possible topics:

5. Developing the number system: natural numbers $\longrightarrow$ integers $\longrightarrow$ rational numbers $\longrightarrow$ real numbers $\longrightarrow$ complex numbers

6. "The Chain Rule is linear algebra"

7. "What lies beyond the Intermediate Value Theorem?" (Answer: the Brouwer Fixed-point Theorem.)

8. "The art of teaching mathematics"

9. "The world of mathematics research" — research problems, journals, refereeing, reviewing, conferences and meetings, and grants

Objective # 2: Learning to talk and write about mathematics.

Tasks directed to this include:

1. Class discussion on "Types of mathematical objects" (2 days) — goal: to become more conscious of the mathematical objects represented by a symbol. Examples: What type of object is $\int \cos(x)\, dx$? (Answer: a *set* of functions.) What type of object is differentiation? (One possible answer: A *function* from the space of differentiable functions from $\mathbb{R}$ to $\mathbb{R}$ to the space of functions from $\mathbb{R}$ to $\mathbb{R}$.)

2. Class discussion on "What is mathematics?" – We tried to come up with a good definition of mathematics. Each student prepared a trial definition and presented it at the board, and the class was supposed to discuss the strengths and weaknesses of each, and then formulate a consensus definition.

3. Class discussion on "A retrospective of my undergraduate mathematics coursework" (2 days)

4. Short individual presentations of "Examples of proofs" illustrating different methods that I had discussed in my lectures on proof.

5. Individual 20-minute seminars on historical mathematical topics (events, ideas, or people). The students' self-selected topics include: early history of topology, early history of number rings, the classification of finite groups, completion of the four-color theorem (two students giving coordinated seminars), development of calculus, life and work of Gödel, Cantor, and Mandelbrot.

6. A written exposition on methods of proof (4 to 10 pages) – I made very little restriction on what students could choose to write on. Most followed my suggested option of reiterating the ideas I discussed in class and finding examples illustrating them, but a few were more creative.

7. (assigned) Longer individual seminars and a written exposition on a self-selected topic (subject to my approval) – The seminar presentations will be the last few weeks of the course, and the written versions are due a week before the end of the semester.

8. (tentative) Class discussion on "My plans after graduation."

9. (tentative) Class discussion on "Mathematics on the internet."

Objective # 3: Mathematics culture

Tasks directed to this include:

1. Read the math department blog

2. Attend Math Club talks (optional)

3. Attend this MAA meeting (optional) - 4 of the students are here

4. Attend Math department colloquia/public lectures (optional), unfortunately there really haven't been any suitable ones this semester

5. Listening to my in-class digressions about mathematics.

What are the challenges?

1. *Student mix.* The students have widely varying levels of experience, self-confidence, and introversion-extroversion.

2. *Class discussions.* The self-confident students tend to dominate, while about half the class seems rather intimidated or at least very reluctant to join in. And my own skills at leading discussions are, should we say, underdeveloped.

3. *Uncharted territory.* Most of my lectures have been the first time I've ever lectured on the topic (How to write math and give talks, LaTeX, the world of research.) I hope to teach the course again next spring, and if I do, I will be able to make some improvements.

4. *Assigning grades.* Most of the criteria on which I want to evaluate student performance are qualitative, not quantitative, and it's hard to decide their relative weights. I'm taking the "stock market" approach (although hopefully less random than the markets): everyone started with a B, and moves up or down (mostly up, in small increments $B+$, $A-/B+$, $A-$) as the course progresses. There is an arbitrary quality to this, and I don't expect all of the students will be pleased, but it seems to me to be the least bad option.

What seems to be working, and what doesn't?

1. In general, the class discussions have not been as lively as I would like. The course retrospective discussion was a flop. Students had little to say, and it was hard to prevent the discussion from veering into a "the things that were wrong with the courses I didn't like" bull session.

2. The one writing assignment so far ("write an exposition about mathematical proof") produced mixed results, although I was generally pleased. Most of the students found it challenging to come up with good examples and especially to relate them to the ideas I had focused on. Almost all of the students had quite a bit of difficulty articulating mathematical ideas, but that is exactly the point of doing this— to develop more skills in this direction.

3. On a more positive note, I think that preparing and presenting a seminar, even a short one, is a very beneficial experience. It's not something math students get to do very much, but the ability to get up and articulate ideas in a mathematical context will be useful in most any math-related career.