Gian-Carlo Rota (1932-1999)

From the 2004 OU MathDay Math Olympics:

Problem: Name a famous mathematician.

College version:

Problem: Name a famous *20*th*-century* mathematician.

One Possible Solution: Gian-Carlo Rota

Rota at work

–Born in Vigevano, Italy in 1932.

–Rota was the son of an anti-Fascist who was condemned to death by Mussolini. The father and his family escaped by crossing over the Alps to Switzerland.

–Rota came to the US in 1950 to be an undergraduate at Princeton, and received his doctorate from Yale in 1956. He became a US citizen in 1961.

–After postdoctoral positions at Courant Institute and Harvard, he went to MIT and was a professor there for the rest of his life. He held many visiting positions, and spent a lot of time at the Los Alamos National Laboratory.

–Published more than 150 articles.

–Supervised 46 doctoral students.

–Received many honors and awards, including the Steele prize in 1988.

–Founded *Journal of Combinatorial Theory* in 1965, and *Advances in Mathematics* in 1967.

All of these are outstanding achievements, but there are quite a few 20^{th}-century mathematicians who had similar accomplishments, yet are not well-known outside of their own research specialities. Why is Rota truly famous in the general mathematical community?

Rota's writing

Rota wrote extensively about mathematics and mathematicians. Many of his essays are collected in the book *Indiscrete Thoughts*, from which I took the passages that I will show you.

A good summary of Rota's writing style is given by Reuben Hirsch in his introduction to *Indiscrete Thoughts*:

> "He loves contradiction. He loves to shock. He loves to simultaneously entertain you and make you uncomfortable."

Review of the book *Sphere Packings, Lattices and Groups,* by J. H. Conway and N. J. A. Sloane:

This is the best survey of the best work in one of the best fields of combinatorics, written by the best people. It will make the best reading by the best students interested in the best mathematics that is now going on.

Review of another book called *Recent philosophers*:

When pygmies cast long shadows, it must be late in the day.

Rota thought deeply about the nature of mathematics itself. We will start with an example of his writing about mathematics, taken from one of his essays. It is characteristically full of surprises, contradictory arguments, and bold statements to challenge our preconceptions.

It concerns the perennial question of whether mathematics is "invented" or "discovered". It also introduces one of Rota's recurring themes: that mathematical advances are eventually refined and abstracted until finally they are seen to be "trivial."

(Note: In his writing, Rota often used very complex syntax, unusual vocabulary, phrases from other languages, and so on. To make the text more suitable for this presentation, I have simplified his original writing in places.)

Are mathematical ideas invented or discovered? This question has been repeatedly posed by philosophers through the ages and will probably be with us forever. We will not be concerned with the answer. What matters is that by asking the question, we acknowledge that mathematics has been leading a double life.

In the first of its lives, mathematics deals with *facts,* like any other science. It is a fact that the altitudes of a triangle meet at a point; it is a fact that there are only seventeen kinds of symmetry in the plane; it is a fact that every finite group of odd order is solvable. The work of a mathematician consists of dealing with such facts in various ways...

In its second life, mathematics deals with *proofs.* A mathematical theory begins with definitions and derives its results from clearly agreed-upon rules of inference. Every fact of mathematics must be put into an axiomatic theory and formally proved if it to be accepted as true. Axiomatic exposition is indispensable in mathematics because the facts of mathematics, unlike the facts of physics, *cannot be experimentally verified.*

We have sketched two seemingly clashing concepts of mathematical truth. Both concepts force themselves upon us when we observe the development of mathematics.

The first concept is similar to the classical concept of the truth of a law of natural science. According to this first view, mathematical theorems are statements of fact; like all facts of science, they are discovered by observation and experimentation. It matters little that the facts of mathematics might be "ideal," while the laws of nature might be "real." Whether real or ideal, the facts of mathematics are out there in the world and are not creations of someone's mind.

The second view seems to lead to the opposite conclusion. Proofs of mathematical theorems, such as the proof of the Prime Number Theorem, are achieved at the cost of great intellectual effort. They are then gradually whittled down to trivialities. Doesn't the process of simplification that transforms a fifty-page proof into a half-page argument support the assertion that theorems of mathematics are creations of our own intellect?

Every mathematical theorem is eventually proved trivial. The mathematician's ideal of truth is triviality, and the community of mathematicians will not cease its beaver-like work on a newly discovered result until it has shown to everyone's satisfaction that all difficulties in the early proofs were merely shortcomings of understanding, and only an analytic triviality is to be found at the end of the road.

Rota thought a lot about different kinds of mathematicians. For example:

Mathematicians can be subdivided into two types: problem solvers and theorizers. Most mathematicans are a mixture of the two, although it is easy to find extreme examples of both types.

To the problem solver, the supreme achievement in mathematics is the solution to a problem that had been given up as hopeless. It matters little that the solution may be clumsy; all that counts is that it should be *first* and that the proof be *correct.* Once the problem solver finds the solution, he will permanently lose interest in it, and will listen to new and simplified proofs with an air of boredom. For him, mathemtics consists of a sequence of challenges to be met, an obstacle course of problems.

To the problem solver, mathematical exposition is regarded as an inferior undertaking. New theories are viewed with deep suspicion, as intruders who must prove their worth by posing challenging problems before they can gain attention. The problem solver resents generalizations, especially those that may succeed in trivializing the solution of one of his problems.

To the theorizer, the supreme achievement of mathematics is a theory that sheds sudden light on some incomprehensible phenomenon. Success in mathematics does not lie in *solving* problems but in their *trivialization.* The moment of glory comes with the discovery of a new theory that does not solve any of the old problems, but renders them irrelevant.

To the theorizer, mathematical concepts received from the past are regarded as imperfect instances of more general ones yet to be discovered. To the theorizer, the only mathematics that will survive are the definitions. Theorems are tolerated as a necessary evil since they play a supporting role— or rather, as the theorizer will reluctantly admit, an essential role— in the understanding of definitions.

If I were a space engineer looking for a mathematician to help me send a rocket into space, I would choose a problem solver. But if I were looking for a mathematician to give a good education to my child, I would unhesitatingly prefer a theorizer.

Rota was very interested in the *teaching* of mathematics. The following passage is from an essay on "beauty in mathematics." Note how Rota connects this topic with the challenges of teaching mathematics, then ends on an unexpectedly dark note:

The beauty of a piece of mathematics is frequently associated with shortness of statement or of proof. How we wish that all beautiful pieces of mathematics shared the snappy immediacy of Picard's theorem. This wish is rarely fulfilled. A great many beautiful arguments are long-winded and require extensive buildup. Familiarity with a huge amount of background material is the condition for understanding mathematics. A proof is viewed as beautiful only after one is made aware of previous clumsier proofs.

Despite the fact that most proofs are long, despite our need for extensive background, we think back to instances of appreciation of mathematical beauty as if they had been perceived in a moment of bliss, in a sudden flash like a light bulb suddenly being lit. The effort put into understanding the proof, the background material, the difficulties encountered in unraveling an intricate sequence of inferences fade and magically disappear the moment we become aware of the beauty of a theorem. The painful process of learning fades from memory and only the flash of insight remains.

We would *like* mathematical beauty to consist of this flash; mathematical beauty *should* be appreciated with the instantaneousness of a light bulb being lit. But it is an error to pretend that the appreciation of mathematical beauty is what we feel it should be, an instantaneous flash. This denial of factual truth occurs much too frequently.

The light bulb mistake is often taken as a paradigm in teaching mathematics. Forgetful of our learning pains, we demand that our students display a flash of understanding with every argument we present. Worse yet, we mislead our students by trying to convince them that such flashes of understanding are the *core* of mathematical appreciation.

Every *good* teacher knows that students will not learn by merely grasping the formal truth of a statement. A mathematical theorem may be *enlightening* or not, just as it may be true or not.

If the statements of mathematics were formally true but in no way enlightening, mathematics would be a curious game played by weird people. Enlightenment is what keeps the mathematical enterprise alive.

Mathematicians seldom acknowledge the phenomenon of enlightenment for at least two reasons. First, unlike truth, enlightenment is not easily formalized. Second, enlightenment admits degrees: some statements are more enlightening than others. Mathematicians dislike concepts admitting degrees, and will go to great lengths to deny the logical role of any such concept. Mathematical beauty is the expression mathematicians have invented in order to obliquely admit the phenomenon of enlightenment while avoiding acknowledgment of the fuzziness of this phenomenon. They say that a theorem is beautiful when what they really mean is that the theorem is enlightening.

The term "mathematical beauty," together with the light bulb mistake, is a trick mathematicians have devised to avoid facing up to the messy phenomenon of enlightenment. It is a copout, one step in a cherished activity of mathematicians, that of building a perfect world immune to the messiness of the ordinary world, a world where what we think *should* be true turns out to *be* true, a world that is free from the disappointment, the ambiguities, the failures of that other world in which we live.

Rota's advice:

1. *Every lecture should make only one main point.*

Every lecture should state one main point and repeat it over and over, like a theme with variations. An audience is like a herd of cows, moving slowly in the direction they are being driven towards. If we make one main point, we have a good chance that the audience will take the right direction; if we make several points, then the cows will scatter all over the field. The audience will lose interest and everyone will go back to the thoughts they interrupted in order to come to the lecture.

2. *Never run overtime.*

Running overtime is the one unforgivable error a lecturer can make. After fifty minutes (one microcentury, as von Neumann used to say) everybody's attention will turn elsewhere even if we are trying to prove the Riemann hypothesis. One minute overtime can destroy the best of lectures.

3. *Give them something to take home.*

I often meet, in airports, in the street and occasionally in embarrassing situations, alumni who have taken one or more courses from me. Most of the time they admit that they have forgotten the subject of the course, and all the mathematics I thought I had taught them. However, they will gladly recall some joke, some anecdote, some quirk, some side remark, or some mistake I made.

4. *Make sure the blackboard is spotless.*

It is particularly important to erase those distracting whirls that are left when we run the eraser over the blackboard in a non-uniform fashion.

By starting with a spotless blackboard, you will subtly convey the impression that the lecture they are about to hear is equally spotless.

5. *Be prepared for old age.*

My late friend Stan Ulam used to remark that his life was sharply divided into two halves. In the first half, he was always the youngest persons in the group; in the second half, he was always the oldest. There was no transitional period.

I now realize how right he was. The etiquette of old age does not seem to have been written up, and we have to learn it the hard way. It depends on a basic realization, which takes time to adjust to. You must realize that, after reaching a certain age, you are no longer viewed as a person. You become an institution, and you are expected to behave like a piece of period furniture or an architectural landmark.

It matters little whether you keep publishing or not. If your papers are no good, they will say, "What did you expect, at his age?" and if an occasional paper of yours is found to be interesting, they will say, "What did you expect? He has been working at this all his life!" The only sensible response is to enjoy playing your newly-found role as an institution.

6. *Never compare fields*

You are not alone in believing that your own field is better than those of your colleagues. We all believe the same about our own fields. Remember, when talking to outsiders, have nothing but praise for your colleagues in all fields, even for those in combinatorics [Rota's field]. All public shows of disunity are ultimately harmful to the well-being of mathematics.

7. *Remember that even the grocery bill is a piece of mathematics*

Once, during a year at a liberal arts college, I was assigned to teach a course on "Mickey Mouse math." I was stung by a colleague's remark that the course "did not deal with real mathematics." But the grocery bill, a computer program, and class field theory are three instances of mathematics.

8. *Do not look down on good teachers*

Mathematics is the greatest undertaking of mankind. All mathematicians know this. Yet many people do not share this view, and consequently most of our income will have to come from teaching. And the more students we teach, the more of our friends we can appoint to our department. Those few colleagues who are successful at teaching undergraduate courses should earn our thanks as well as our respect. It is counterproductive to turn up our noses at those who bring home the dough.

When Mr. Smith dies and decides to leave his fortune to our mathematics department, it will be because he remembers his good teacher Dr. Jones, not because of all the research papers you have written.