MATH 1523: Quiz #1

Show every step of your work. Put a box around your final answer.

1. Rationalize the denominator: [20 pts]

\[
\frac{4}{\sqrt{11} - \sqrt{7}} \left( \frac{\sqrt{11} + \sqrt{7}}{\sqrt{11} + \sqrt{7}} \right) = \frac{4(\sqrt{11} + \sqrt{7})}{11 - 7} = \frac{4(\sqrt{11} + \sqrt{7})}{4} = \sqrt{11} + \sqrt{7}
\]

2. Factor the polynomial: [20 pts]

\[b^3 - 8a^6 = (b)^3 - (2a^2)^3\], which is a difference of cubes!

Plug \(X = b\) and \(Y = 2a^2\) into the difference of cubes formula: \(X^3 - Y^3 = (X - Y)(X^2 + XY + Y^2)\)

\[b^3 - 8a^6 = (b)^3 - (2a^2)^3 = (b - 2a^2)(b^2 + 2a^2b + 4a^4)\]

3. Solve the following inequality: [20 pts]

\[|3 - 2x| \leq 5\]

\[3 - 2x \leq 5 \quad 3 - 2x \geq -5\]
\[-2x \leq 2 \quad -2x \geq -8\]
\[x \geq -1 \quad x \leq 4\]

\[x \geq -1 \text{ and } x \leq 4\]

\([-1, 4]\]

4. A rectangular room has area of 143 square feet. If the length is 2 feet longer than the width, find the dimensions of the room. [20 pts]

\[143 = lw = (w + 2)w\]
\[143 = w^2 + 2w\]
\[0 = w^2 + 2w - 143\]
\[0 = (w - 11)(w + 13)\]
\[w = 11, \quad w = -13\]

However, a negative length is impossible, so \(w = 11 \Rightarrow l = 11 + 2 = 13\)

5. Simplify: [20 pts]

\[
\frac{x+1}{x^2-9x+8} - \frac{3x}{2x^2-5x+3} = \frac{x+1}{(x-8)(x-1)} - \frac{3x}{(2x-3)(x-1)} = \frac{(x+1)(2x-3) - (3x)(x-8)}{(x-8)(x-1)(2x-3)} = \frac{2x^2 - 3x + 2x - 3 - 3x^2 + 24}{(x-8)(x-1)(2x-3)} = \frac{-x^2 + 23x - 3}{(x-8)(x-1)(2x-3)}
\]