H.W. 3

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1) As x approaches 2, f(x) approaches 5.

Yes, the graph could have a hole at (2,5)

and be defined such that $f(2) = 3$

3) a) $\lim_{x \to -3} f(x) = \infty$ means that the values of

f(x) can be made arbitrarily large by taking

x sufficiently close to -3

b) $\lim_{x \to 4^+} f(x) = -\infty$ means that the values of

f(x) can be made arbitrarily large nega-

tive by taking x sufficiently close to

4 through values larger than 4.

4) a) $\lim_{x\to 0} f(x) = 3$ b) $\lim_{x\to 3^-} f(x) = 4$ c) $\lim_{x\to 3^+} f(x) =$

d) $\lim_{x\to 3} f(x)$ DNE e) $f(3) = 3$

23) $\lim_{x\to 5^+} \frac{6}{x-5} = ?$ where $f(x) = \frac{6}{x-5}$.

Sol: Let $x = 5.1$ then $\frac{6}{5.1-5} = 60$

$x = 5.01$ then $\frac{6}{5.01-5} = 600$

$x = 5.001$ then $\frac{6}{5.001-5} = 6000$ $\downarrow \infty$

For small enough x values $\lim_{x\to 5^+} f(x) = \infty$

24) $\lim_{x\to 5^-} \frac{6}{x-5} = ?$

Sol: Let $x = 4.9$, $\frac{6}{4.9-5} = -60$

$x = 4.99$, $\frac{6}{4.99-5} = -600$ $\downarrow -\infty$

$\vdots$

hence $\lim_{x\to 5^-} \frac{6}{x-5} = -\infty$

25) $\lim_{x \to 1} \frac{2-x}{(x-1)^2} = ?$

Sol: Since the numerator is positive and the denominator approaches 0 through positive values as $x \to 1$.

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14) $\lim_{x \to 4} \frac{x^2-4x}{x^2-3x-4} = \lim_{x \to 4} \frac{x.(x-4)}{(x-4)(x+1)} = \lim_{x \to 4} \frac{x}{x+1} = \frac{4}{5}$

18) $\lim_{x \to 1} \frac{x^3-1}{x^2-1} = \lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = \lim_{x \to 1} \frac{x^2+x+1}{x+1} =$

20) $\lim_{h \to 0} \frac{(2+h)^3-8}{h} = \lim_{h \to 0} \frac{(8+12h+6h^2+h^3)-8}{h}$

$= \lim_{h \to 0} \frac{h^3+6h^2+12h}{h}$

$= \lim_{h \to 0} \frac{h(h^2+6h+12)}{h}$

$= \lim_{h \to 0} h^2+6h+12 = 12$

30) $\lim\limits_{x\to 1} \dfrac{\sqrt{x}-x^2}{1-\sqrt{x}} = \lim\limits_{x\to 1} \dfrac{\sqrt{x}\,(1-x^{3/2})}{1-\sqrt{x}}$

$= \lim\limits_{x\to 1} \dfrac{\sqrt{x}\,(1^3-(x^{1/2})^3)}{1-\sqrt{x}}$

$= \lim\limits_{x\to 1} \dfrac{\sqrt{x}\,\cancel{(1-\sqrt{x})}\,(1+\sqrt{x}+x)}{\cancel{1-\sqrt{x}}}$

$= \sqrt{1}\cdot(1+\sqrt{1}+1)$

$= 3$

37) Prove that $\lim\limits_{x\to 0} x^4 \cdot \cos\left(\frac{2}{x}\right) = 0$

<u>proof</u>: $-1 \leq \cos\left(\frac{2}{x}\right) \leq 1$

$\Rightarrow -x^4 \leq x^4 \cdot \cos\left(\frac{2}{x}\right) \leq x^4$

Since $\lim\limits_{x\to 0}(-x^4) = 0$ and $\lim\limits_{x\to 0} x^4 = 0$

by the Squeeze thm. $\lim\limits_{x\to 0} x^4 \cdot \cos\left(\frac{2}{x}\right) = 0$.

56) Let $f(x) = [\![x]\!]$ and $g(x) = -[\![x]\!]$. Then $\lim_{x \to 3} f(x)$ and $\lim_{x \to 3} g(x)$ do not exist but

$$\lim_{x \to 3} [f(x) + g(x)] = \lim_{x \to 3} ([\![x]\!] - [\![x]\!]) = \lim_{x \to 3} 0 = 0$$