Hw #4

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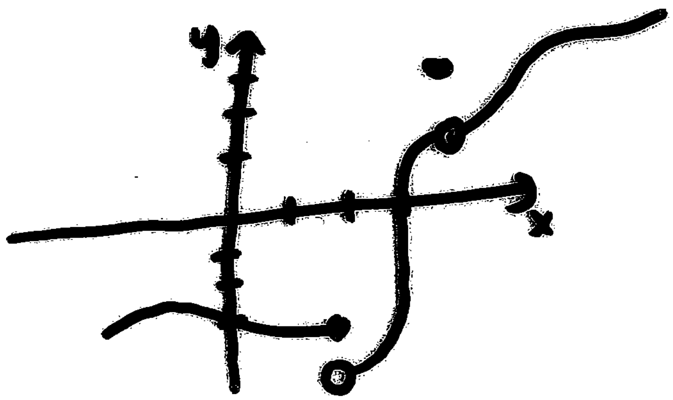
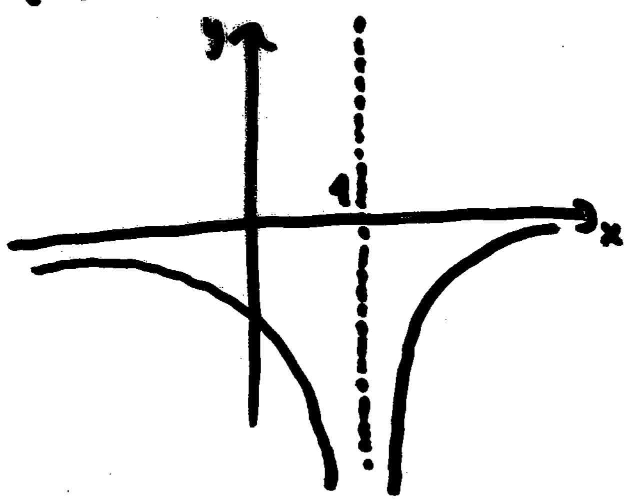
6) The graph can be shown as;



(The answer vary)

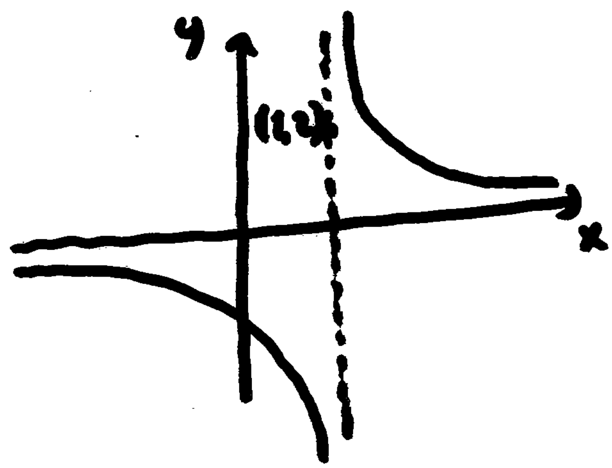
15) $f(x) = -\dfrac{1}{(x-1)^2}$ is discontinuous at 1

since $f(1)$ is not defined.



16) $f(x) = \begin{cases} \dfrac{1}{x-1} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1 \end{cases}$ is discontinuou

at 1 because $\lim_{x \to 1} f(x)$ does not exist.



$\lim_{x \to 1^+} f(x) = \infty$

$\lim_{x \to 1^-} f(x) = -\infty$

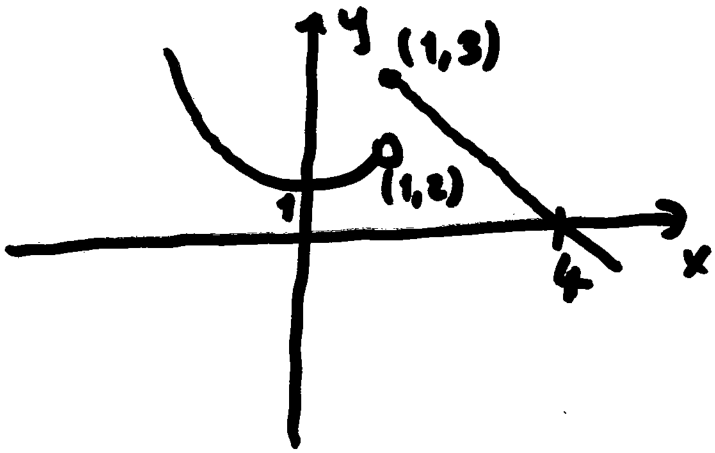
20) $f(x) = \begin{cases} 1 + x^2 & \text{if } x < 1 \\ 4 - x & \text{if } x \geqslant 1 \end{cases}$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (1 + x^2) = 1 + 1 = 2$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (4 - x) = 4 - 1 = 3$

Thus, f is dicontinuous at 1 because

$\lim_{x \to 1} f(x)$ does not exist.



24) By theorem 7, the trigonometric function $\sin x$ and the polynomial function $x+1$ are continuous on $\mathbb{R}$. By part 5 of theorem 4, $h(x) = \frac{\sin x}{x+1}$ is continuous on its domain

$\{x \mid x \neq -1\}$

28) The sine and cosine functions are continuous everywhere by theorem 7 so

$$F(x) = \sin(\cos(\sin x))$$

is continuous everywhere by theorem 9.

43) $f(x) = x^3 - x^2 + x$ is continuous on the interval $[2,3]$ since it is a polynomial.

$f(2) = 6$ and $f(3) = 21$.

Since $6 < 10 < 21$, there is a number c in $(2,3)$ such that $f(c) = 10$ by Intermediate Value Theorem

59) If there is such a number, it satisfies the equation $x^3 + 1 = x$

$$\Leftrightarrow x^3 - x + 1 = 0$$

Let $f(x) = x^3 - x + 1$

$f(x)$ is continuous since it is a polynomial and $f(-2) = -5 < 0$ and $f(-1) = 1 > 0$. Now by Intermediate Value theorem, there is a number c between -2 and -1 such that $f(c)=0$, so that $c^3+1=c$.

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8) Using $\lim\limits_{x\to x_0} \frac{f(x)-f(x_0)}{x-x_0} = m$,

$$m = \lim_{x\to 4} \frac{\sqrt{2x+1}-\sqrt{2.4+1}}{x-4} = \lim_{x\to 4} \frac{\sqrt{2x+1}-3}{x-4}$$

$$= \lim_{x\to 4} \frac{\sqrt{2x+1}-3}{x-4} \cdot \frac{(\sqrt{2x+1}+3)}{(\sqrt{2x+1}+3)}$$

$$= \lim_{x\to 4} \frac{2x+1-3^2}{(x-4)(\sqrt{2x+1}+3)}$$

$$= \lim_{x\to 4} \frac{2\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{2x+1}+3)} = \frac{2}{\sqrt{2.4+1}+3}$$

$$= \frac{1}{3}$$

Tangent line: $y-3 = \frac{1}{3}(x-4)$

$$y = \frac{1}{3}(x-4)+3$$

15) a) Since the slope of the tangent at $t=0$ is 0, the car's initial velocity was 0.

b) The slope of the tangent is greater at C than at B, so the car was going faster at C.

c) Near A, the tangent lines are becoming steeper as x increases, so the velocity was increasing, so the car was speeding up.

Near B, the tangent lines are becoming less steep, so the car was slowing down.

The steepest tangent near C is the one at C, so at C the car had just finished speeding up, and was about to start slowing down.

18) a) $$v(1) = \lim_{h \to 0} \frac{H(1+h) - H(1)}{h}$$

$$= \lim_{h \to 0} \frac{(58 + 58h - 0.83 - 1.66h - 0.83h^2) - 57.17}{h}$$

$$= \lim_{h \to 0} (56.34 - 0.83h) = 56.34 \text{ m/s}.$$

b) $$v(a) = \lim_{h \to 0} \frac{H(a+h) - H(a)}{h}$$

$$= \lim_{h \to 0} \frac{(58a + 58h - 0.83a^2 - 1.66ah - 0.83h^2) - H(a)}{h}$$

$$= \lim_{h \to 0} (58 - 1.66a - 0.83h) = 58 - 1.66a \text{ m/s}$$

c) The arrow strikes the moon when the height is 0, that is

$$58t - 0.83t^2 = 0$$

$$t(58 - 0.83t) = 0$$

hence $t = \frac{58}{0.83} \approx 69.9$ s (since $t \neq 0$)

d) From part (c), $v\left(\frac{58}{0.83}\right) = 58 - 1.66 \cdot \frac{58}{0.83} = -58$ m/s

Thus, the arrow will have velocity of -58 m/s.