MATH 2513-001 | Discrete Mathematics | Fall 2005

Questions on Cardinality of Infinite Sets

1. Prove that $[0,1)$ and $(0,1)$ have the same cardinality.
2. Prove that $(0,1)^3$ (which is a subset of $\mathbb{R}^3$) and $(0,1)$ have the same cardinality.
3. Consider the bijection $f : (0,1)^2 \to (0,1)$ described in class notes. Show that if $x, y \in \mathbb{Q}$ then $f(x,y) \in \mathbb{Q}$.
4. What about the converse to the question above? If $f(x,y) \in \mathbb{Q}$ do x and y have to be rational?
5. Write the following fractions out in base 3, without using the digit 1 in your base 3 expansion.
$$\frac{1}{3} \qquad \frac{10}{27} \qquad \frac{1}{4} \qquad \frac{3}{4}$$
6. We saw in class that the base 3 expansion of a number which does not involve the digit 1, gives a bijection between the Cantor set, C, and the power set $\mathcal{P}(\mathbb{Z}^+)$. What can you say about one of the endpoints of an interval in A_n (is it rational or irrational?, why?)? Argue that there are only countably many such endpoints?
7. The previous question shows that there must be more elements in C. Are there rational numbers in C which are different from the endpoints of one of the intervals in A_n for some n?
8. Find an explicit irrational number in C? Say why it is irrational! [Hint: use base 3 expansions. Remember that a rational number has a terminating or repeating pattern decimal expansion. Is the same true for base 3? How might this help you look for irrational elements of C?]