Friday 05/07/2004 Final Examination 10:30am–12:30pm

Name: Student ID:

Instructions.

1. No calculators or notes.
2. Attempt all questions.
3. Do not write on back of exam sheets. Extra paper is available.
4. Show all the steps of your work clearly.
   The method (reasoning) used to obtain an answer is worth more than the answer itself.

Question	Points	Your Score
Q1	4	
Q2	8	
Q3	9	
Q4	9	
Q5	9	
Q6	18	
Q7	9	
Q8	9	
TOTAL	75	

Q1]. . . [4 points] Compute the derivative $\frac{dy}{dx}$ of the following function.

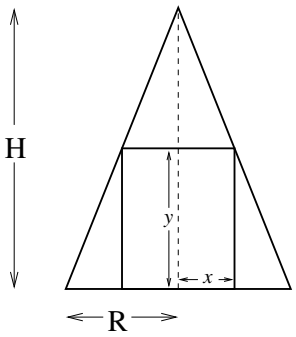
$$y = \sqrt{x^2 + 1}$$

Q2]. . . [8 points] Show that the two curves $x^2 + 2y^2 = 2$ and $2x^2 - 2y^2 = 1$ intersect orthogonally (perpendicularly).

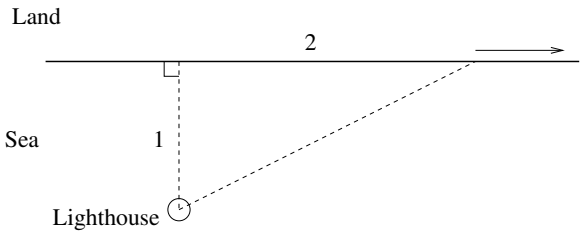
Q3]. . . [9 points] State the Mean Value Theorem.

Use the Mean Value Theorem to prove that if $f(x)$ is a differentiable function, and $f'(x) > 0$ for all points x on some interval, then $f(x)$ is increasing on this interval.

Q4]. . . [9 points] Find the maximum value of the curved surface area of a right circular cylinder which can be inscribed inside a right circular cone of base radius R and height H (as shown). You may recall that the curved surface area of a right circular cylinder with base radius x and height y is given by $2\pi xy$.



Q5]. . . [9 points] A beam of light from a lighthouse rotates at 2 radians/minute. The lighthouse is located 1 mile off shore as shown. How fast is the point of light moving along the shoreline when it is 2 miles from the closest point on the shore to the lighthouse? Give your answer in miles/minute.



Q6]. . . [18 points] Consider the function

$$f(x) = \frac{x}{x^2+1}$$

Compute the following.

1. $f'(x)$. _____
2. $f''(x)$. _____
3. Intervals where f is increasing. _____
4. Intervals where f is decreasing. _____
5. Intervals where f is concave up. _____
6. Intervals where f is concave down. _____
7. Critical points; (local max/min, neither). _____
8. Points of inflection. _____
9. x- and y-intercepts. _____
10. Horizontal asymptote. _____
11. Draw the graph of $f(x)$ which displays all the information above. There is an extra page for this problem.

Extra space for Q6.

Q7]. . . [9 points] Write down the general formula in Newton's method for finding approximations to the roots of $f(x) = 0$.

Use Newton's method to derive a formula for approximating $\sqrt{6}$. Starting with an initial guess of $x_1 = 2$, compute x_2 and x_3.

Q8]. . . [9 points] Use anti-differentiation to find $f(x)$. You are given the following three pieces of information.

$$f''(x) = \cos(x) + 2$$
$$f'(0) = 0$$
$$f(0) = 0$$