

Q1].. True or False.

- (1) If $\mathbf{T}(t)$ is the unit tangent vector to a smooth curve in 3-dimensions, then the curvature κ is given by

$$\kappa = \left| \frac{d\mathbf{T}}{dt} \right|$$

FALSE — only works if $t = s$ (arc length). ^{For example,} if you travel along path ~~(at twice)~~ so $t = 2s$, then $\kappa = 2 \left| \frac{d\hat{\mathbf{T}}}{dt} \right|$.

- (2) If the acceleration of a particle is always parallel to the position $\mathbf{r}(t)$ of the particle, then $\left(\vec{v} = \text{velocity} \right)$

$$\frac{d}{dt}(\mathbf{r} \times \mathbf{v}) = 0$$

TRUE

$$\frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt} = \vec{v} \times \vec{v} + \vec{r} \times \vec{a} = \vec{0} + \vec{0} = \vec{0}$$

- (3) If $|\mathbf{r}(t)| = 1$ for all t , then $\mathbf{r}'(t)$ is perpendicular to $\mathbf{r}(t)$.

TRUE

$$\vec{r}(t) \cdot \vec{r}(t) = 1 \Rightarrow 2 \vec{r}(t) \cdot \vec{r}'(t) = \frac{d(1)}{dt} = 0 \Rightarrow \vec{r}(t) \perp \vec{r}'(t)$$

- (4) If $|\mathbf{r}(t)| = 1$ for all t , then $|\mathbf{r}'(t)| = 1$ for all t .

eg $\vec{r}(t) = \langle \cos(50t), \sin(50t) \rangle, |\mathbf{r}(t)| = 1$

FALSE

$$\left| \frac{d\mathbf{r}}{dt} \right| = 50$$