Fa'09: MATH 2513–001 **Discrete Mathematics** **Noel Brady**

Tuesday 11/24/2009 Midterm III 9:00am–10:15am

Name: Student ID:

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	20	
Q2	20	
Q3	24	
Q4	36	
TOTAL	100	

Q1]. . . [20 points] Define what it means for a function $f : A \to B$ to be surjective.

Recall that $(0, 1)$ denotes the interval $\{x \in \mathbb{R} \mid 0 < x < 1\}$, and that $\mathbb{Z}^+$ denotes the set of positive integers. Give a detailed proof that no function

$$f : \mathbb{Z}^+ \to (0, 1)$$

can be surjective. (This is the usual Cantor diagonalization argument that $(0, 1)$ is uncountable).

Q2]. . . [20 points] Prove that there is a bijection between the set $\mathbb{Z}$ and the set $\mathbb{Z}^+$.

Prove that there is a bijection between the set $\mathbb{Z}^+ \times \mathbb{Z}^+$ and the set $\mathbb{Z}^+$.

Q3]. . . [24 points] Define what it means for the set A to be countable.

Define what it means for the set A to be uncountable.

Say whether each of the following sets is countable or not.

1. The set $\mathbb{Z}$ of all integers.
2. The set $\mathbb{R}$ of all real numbers.
3. The set $\mathbb{Q}$ of all rational numbers.
4. The set of all irrational numbers.
5. The set of all functions from $\{1\}$ to $\mathbb{R}$.
6. The set of all functions from $\mathbb{R}$ to $\{1\}$.
7. The set of all functions from $\{1, 2, 3\}$ to $\mathbb{Z}$.
8. The set of all functions from $\mathbb{Z}$ to $\{1, 2, 3\}$.
9. The set of all lines in the plane $\mathbb{R}^2$.
10. The power set $\mathbb{P}(\mathbb{Z})$ of $\mathbb{Z}$.

Q4]. . . [36 points] True or False.

1. The composition of reflections in two intersecting lines is a rotation.

2. The set of symmetries of a regular pentagon (5 sides) has 10 elements.

3. The set of symmetries of a regular polygon with 1,000 sides is countable.

4. The set of symmetries of a circle is countable.

5. Perm($\{1, 2, \ldots, n\}$) has n^n elements.

6. Perm($\mathbb{Z}^+$) is countable.

7. $(123)(245)(132) = (345)$.

8. If m is reflection in some line, and R is a 90° counterclockwise rotation about a point O, then mRm is a 90° counterclockwise rotation about the point $m(O)$.

9. $(12)(23)(34)(45)(56)(67)(78) = (12345678)$

10. $(12)(23)(34)(45)(34)(23)(12) = (15)$.

11. The composition of reflections in the three sides of a triangle (taken in any order) is a rotation.

12. The composition of reflections in the four sides of a rectangle (taken in any order) is a translation.