

Thursday 12/15/2005

Final Examination

1:30pm-3:30pm

Name: Student ID: **Instructions.**

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	15	
Q2	10	
Q3	10	
Q4	10	
Q5	10	
Q6	15	
Q7	15	
Q8	15	
TOTAL	100	

Q1]... [15 points] Write down the limit definition of the derivative of the function $f(x)$ at the point a .

Use the limit definition to compute the derivative of the function $f(x) = x^2$. [Do not use power rule!]

Compute the following limit

$$\lim_{x \rightarrow \pi/6} \frac{\cos(2x) - (1/2)}{x - (\pi/6)}$$

Hint: Think about limit definitions of derivatives!

Q2]... [10 points] State the Mean Value Theorem.

Use the Mean Value Theorem to give a proof of the following fact:
If $f'(x) > 0$ for all inputs x , then $f(x)$ is an increasing function.

Q3]... [10 points] Compute the following higher derivatives.

1. $f^{(100)}(x)$ where $f(x) = 19x^{54} + 2x^{25} - 13x + 4$.

2. $f^{(137)}(x)$ where $f(x) = \sin(3x + 2)$.

Q4]... [10 points] Find the equation of the tangent line to the implicit curve

$$x + 2y + 1 = \frac{y^2}{x - 1}$$

at the point $(2, -1)$.

Q5]... [10 points] A police officer standing near a highway is using a radar gun to catch speeders. The officer aims the radar gun at a car that has just passed by and, when the radar gun is pointing at an angle of $\pi/4$ to the direction of the highway, the officer notes that the distance between the car and the radar gun is increasing at a rate of 100km/hr. Compute the speed of the car at that instant?

Q6]... [15 points] Find the maximum volume of a right-circular cone which can be inscribed inside of a sphere of radius R . Recall that the volume of a right-circular cone is $(1/3)\pi r^2 h$ where r is the radius of the base of the cone, and h is the height of the cone.

Q7]... [15 points] Consider the function

$$f(x) = \frac{x^2}{x^2 + 1}$$

Answer the following questions about $f(x)$ and **draw the graph** of $f(x)$.

1. Determine the x - and y -intercepts of f .
2. Does the graph of $f(x)$ have any symmetry?
3. Does the graph of $f(x)$ have any vertical asymptotes?
4. Does the graph of $f(x)$ have any horizontal asymptotes?
5. Compute $f'(x)$.

6. Find the critical points, intervals where f is increasing/decreasing and local max/min for f .

7. Compute $f''(x)$.

8. Find intervals where f is concave up and where f is concave down, and inflection points.

Sketch the graph of $f(x)$ indicating the information above.

Q8]... [15 points] Compute the derivatives $f'(x)$ of the following three functions.

1. $f(x) = \sin^2(3x^3 - 4)$

2. $f(x) = (\sqrt{x - 1}) \tan(x)$

3. $f(x) = \frac{(x^2+2)}{(x-2)^2}$