$10 \equiv 1 \pmod 3$ Definition of an $(n+1)$ digit integer

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$\Rightarrow \quad a_n \_\_\_ a_1 a_0 = a_n(10^n) + \cdots + a_1(10^1) + a_0$

$\equiv a_n(1^n) + \cdots + a_1(1^1) + a_0 \pmod 3$

$\equiv (a_n + \cdots + a_0) \pmod 3$

So $3 \mid (LHS) \Leftrightarrow LHS \equiv 0 \pmod 3$

$\Leftrightarrow RHS \equiv 0 \pmod 3$

$\Leftrightarrow 3 \mid (RHS)$

Prove Exact same rule holds for divisibility by 9.

Divisibility by 3

The $(n+1)$ digit number $a_n \_\_\_ a_1 a_0$ is divisible by 3 if and only if the sum of its digits $a_n + \cdots + a_0$ is divisible by 3

eg $3 \nmid (1+2+3+4+5+6+7) \Rightarrow 3 \nmid 1{,}234{,}567$ $3 \nmid 28$

$3 \mid 27$ so if we change last digit to 6 then

$3 \mid 1{,}234{,}566$

for example.

$$10 \equiv -1 \pmod{11}$$

$11 = 10-(-1)$
$\Rightarrow 11 \mid (10-(-1))$
$\Rightarrow 10 \equiv (-1) \pmod{11}$

Thus the $(n+1)$-digit number

By definition of $(n+1)$ digit number

$$a_n \text{———} a_1 a_0 = a_n(10^n) + \cdots + a_1(10^1) + a_0$$

$$\equiv a_n(-1)^n + \cdots + a_1(-1) + a_0 \pmod{11}$$

So $11 \mid \text{LHS} \iff \text{LHS} \equiv 0 \pmod{11}$
$\iff \text{RHS} \equiv 0 \pmod{11} \iff 11 \mid (\text{RHS})$

Divisibility by 11

The $(n+1)$-digit number $a_n \text{———} a_1 a_0$ is divisible by 11 if and only if the alternating signed sum of its digits

$$a_0 - a_1 + a_2 - \cdots + (-1)^n a_n$$

is divisible by 11.

eg $11 \nmid ((7-6)+(5-4)+(3-2)+1)$ since $11 \nmid 4$

Thus $11 \nmid 1{,}234{,}567$.

ok if we subtract 4 from units digit, a_0.

But $11 \mid 1{,}234{,}563$