(1)

Let $S = \{ n \in \mathbb{N} \mid n \text{ is a multiple of } 6 \}$

$T = \{ n \in \mathbb{N} \mid n \text{ is even} \}$

Prove $S \subseteq T$

Proof

Let $n \in S$

By definition of S, n is a multiple of 6.

This means $n = 6q$ for some $q \in \mathbb{Z}$.

This means $n = 2(3q)$ where $3q \in \mathbb{Z}$

This means n is even.

Thus $n \in T$. by def^n of T.

We have shown: $(n \in S) \longrightarrow (n \in T)$

Thus $S \subseteq T$.

$\boxtimes$

Let $A = \{ n \in \mathbb{Z} \mid n \equiv 3 \pmod{12} \}$

$B = \{ m \in \mathbb{Z} \mid m \equiv 2 \pmod{8} \}$

Prove : $A \cap B = \emptyset$

Proof We argue by contradiction. Suppose $A \cap B \neq \emptyset$. Therefore $A \cap B$ contains an integer n.

$n \in A \Rightarrow n \equiv 3 \pmod{12}$

$\Rightarrow n = 3 + 12q$ for some integer q

$\Rightarrow n =$ odd + even

$\Rightarrow n$ is odd.

$n \in B \Rightarrow n \equiv 2 \pmod{8}$

$\Rightarrow n = 2 + 8p$ for some $p \in \mathbb{Z}$

$\Rightarrow n = 2(1 + 4p)$

$\Rightarrow n$ is even.

We have a contradiction: "n is odd" and "n is even".

Thus $A \cap B \neq \emptyset$ is false, & so $A \cap B = \emptyset$ ∎