PRINT NAME: _____

Calculus III [2433–001] Midterm II

For full credit, give reasons for all your answers.

Q1]... In this question we will analyze the *asteroid* parametric curve

$$x = \cos^3 t \qquad\qquad y = \sin^3 t\,.$$

You will be asked to draw a picture of this curve on the next page.

(a) Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Say where this curve is increasing/decreasing, and where it is concave up/concave down. Also, find the points where this curve has horizontal or vertical tangent directions.

Sketch a picture of the asteroid curve.

Q2]... Compute the length of the following parametric curve

$$x = \sin^2 t \qquad y = 2\cos t \qquad 0 \le t \le \pi/2$$

[You may need to look up a table of integrals here]

Q3]... Use the Binomial Series to help you find a power series (in x) for the function $f(x) = (8 - x^2)^{1/3}$.
What is the radius of convergence of the resulting series?

Q4]... Compute the Taylor series for $\ln(x)$ about the point 2.
What is the radius of convergence of this series?
What does setting $x = 1$ in this series tell you about $\ln 2$?