PRINT NAME: Solutions

Calculus IV [2443–002] Quiz III

Tuesday, April 4, 2000

Q1]... Write the following triple integral out as a spherical coordinates triple integral.

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} z(x^2+y^2+z^2)dzdydx$$

Soln: The region is precisely one quarter of a solid ball which is centered on the origin and has radius 3. The quarter is is above the xy-plane and to the positive y half of the xz-plane.

The spherical coordinates description of this region is just

$$0 \le \rho \le 3\,, \qquad 0 \le \theta \le \pi\,, \qquad 0 \le \phi \le \pi/2\,.$$

Noting that the integrand converts into $\rho\cos\phi(\rho^2)$, and remembering that $dv = \rho^2 \sin\phi d\rho d\theta d\phi$, we obtain

$$\int_{0}^{\pi/2}\int_{0}^{\pi}\int_{0}^{3} \rho^5 \cos\phi \sin\phi d\rho d\theta d\phi\,.$$

Q2]... Sketch the region which is described in the following triple integral.

$$\int_{0}^{\pi/4}\int_{0}^{\pi/2}\int_{0}^{\sec\phi} \rho^2 \sin\phi d\rho d\theta d\phi$$

Soln: We build the region up from small blocks which radiate outwards from the origin ($\rho = 0$) until the horizontal plane $z = 1$ ($\rho = \sec\phi$). We see this last fact from the definition of spherical coordinates as follows: $\rho = \sec\phi \Rightarrow \rho\cos\phi = 1 \Rightarrow z = 1$. These blocks produce a type of cone with vertex at the origin and base on the plane $z = 1$. Now we rotate these cones one quarter way about the z axis – from the positive x-axis ($\theta = 0$) until the positive y-axis ($\theta = \pi/2$). Finally, we build up copies of this region from the vertical ($\phi = 0$) until an angle of $\pi/4$ with the z-axis ($\phi = \pi/4$).

Our resulting region is one quarter of a solid cone of height 1 and cone angle (between axis and side of cone) of $\pi/4$. The cone has vertex at the origin, and "base" on the pane $z = 1$. The quarter corresponds to the first octant $x \ge 0, y \ge 0, z \ge 0$.