Math 2443-002 **Conservative Vector Fields** **Spring 2000**

$\mathbf{F} = \langle P, Q \rangle$ where P and Q have continuous first partial derivatives on a domain D in $\mathbf{R}^2$, and

$$\boxed{P_y = Q_x}$$

on all of D

$\mathbf{F} = \langle P, Q, R \rangle$ where P, Q and R have continuous first partial derivatives on a domain D in $\mathbf{R}^3$, and

$$\boxed{P_y = Q_x\,, \quad P_z = R_x\,, \quad Q_z = R_y}$$

on all of D

Green's Thm
(simply connected D)

Set $P = f_x$ etc
and just compute

Stokes' Thm
(eg. $D = \mathbf{R}^3$)

$\int_C \mathbf{F} \cdot d\mathbf{r} = 0$
for every closed path
in D

$\int_C \mathbf{F} \cdot d\mathbf{r}$
is independent of the
path C between two
points of D

I.. Pick a basepoint.
II.. Use $\int_C \mathbf{F} \cdot d\mathbf{r}$ in order to define a function f.
III.. Verify that $\nabla f = \mathbf{F}$.

The Fundamental Theorem:
$\mathbf{F} = \nabla f$
implies that
$\int_C \mathbf{F} \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$

F is **conservative**
means

$$\boxed{\mathbf{F} = \nabla f}$$

where f is some
function defined on D