Honors Problem Set I Riemann Sum Examples

Q1. Prove that

$$\int_a^b x^2\,dx \;=\; \frac{b^3-a^3}{3}$$

using limits of Riemann sums. Use equal width subintervals and right hand endpoints as evaluation points.

Q2. Use Riemann sums with equal width subintervals and right hand endpoints to evaluate the definite integral

$$\int_a^b \sin(x)\,dx$$

(a) Let $h = (b-a)/n$. Show that the Riemann sum is

$$h\sum_{i=1}^{n} \sin(a+ih)$$

(b) We can multiply and divide this expression by $2\sin(h/2)$ provided that h is not an integer multiple of 2π (why this condition?).

Now use the trig identity (why is this identity true?)

$$2\sin(A)\sin(B) \;=\; \cos(A-B) - \cos(A+B)$$

in our expression. Something wonderful happens and the Riemann sum collapses down to

$$\frac{h}{2\sin(h/2)}\left(\cos(a+h/2) - \cos(b+h/2)\right)$$

Fill in the details!

(c) Now take the limit of these Riemann sums as $n \to \infty$. You may find the classical limit (which we recall from Calc I is of central importance in the differentiation of trig functions)

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} \;=\; 1$$

to be useful.

Q3. How would you modify the steps outlined above to evaluate the following definite integral? Work through the steps in this case. Show all your work carefully.

$$\int_a^b \cos(x)\,dx$$

Q4. Here's a situation where it's is easier when we do **not** use endpoints in evaluating the Riemann sums. Work problem **71** on page 312 of the textbook.