Thursday 05/08/2008 Final Examination 120 minutes

Name: ______________________ Student ID: ______________

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.
4. No calculators, no notes, no books, no cell phones.

Question	Points	Your Score
Q1	10	
Q2	10	
Q3	10	
Q4	10	
Q5	10	
Q6	15	
Q7	10	
Q8	15	
Q9	10	
TOTAL	100	

Q1]... [10 points] Evaluate the following limit by first converting it into a suitable form for l'Hospital's rule.

$$\lim_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$$

Q2]. . . [10 points] Evaluate the indefinite integral

$$\int \sin^4(x)\,dx$$

Q3]. . . [10 points] Evaluate the following improper integral

$$\int_e^{\infty} \frac{\ln(x)dx}{x^{10}}$$

Q4]. . . [10 points] Find an expression for the inverse of the following function

$$y = \frac{10^x - 10^{-x}}{2}$$

Q5]... [10 points] Using the SHELL method, determine the volume (of the "bagel") obtained by rotating the region inside the circle

$$(x-2)^2 + y^2 = 1$$

about the y-axis.

Q6]. . . [15 points] Determine the surface area (of the "bagel") obtained by rotating the circle

$$(x-2)^2 + y^2 \;=\; 1$$

about the y-axis.

Q7]... [10 points] Write down the work done when the constant force F lbs moves an object through a distance d ft.

A uniform 50 ft long rope weighing 2 lb/ft hangs over the edge of a tall building as shown. Using Riemann sums, show how to obtain an integral expression for the work done in lifting the rope to the top of the building. Give reasons for your steps.

What is the work done in lifting the rope above to the top of the building? That is, evaluate the integral you found above.

Q8]. . . [15 points] Find the values of A, B and C which make the following (partial fractions) equation true.

$$\frac{1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

Now, evaluate the integral

$$\int \frac{dx}{(x-1)(x^2+1)}$$

Q9]. . . [10 points] Determine if the function

$$F(x) \;=\; \int_0^{2x-x^2} \cos\left(\frac{1}{1+t^2}\right)\, dt$$

has any maximum or a minimum values?

1. Trig Addition, Half Angle.

   $\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$
   $\cos(2A) = \cos^2(A) - \sin^2(A)$
   $\cos(2A) = 2\cos^2(A) - 1$
   $\cos(2A) = 1 - 2\sin^2(A)$
   $\sin^2(x) = (1 - \cos(2x))/2$
   $\cos^2(x) = (1 + \cos(2x))/2$
   $\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
   $\sin(2x) = 2\sin(x)\cos(x)$.

2. Hyperbolic.

   $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$
   $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$

3. Integration by Parts.

   $\int u\,dv \;=\; uv \;-\; \int v\,du$

4. Inverse Trig.

   $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$
   $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$
   $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}(\frac{x}{a})$

5. Jon McCammond Dictionary.

   $u = \sec(x) + \tan(x)$ and $v = \sec(x) - \tan(x)$
   $\sec(x) = \frac{u+v}{2}$, and $\tan(x) = \frac{u-v}{2}$, and $uv = 1$
   $\sec(x)dx = \frac{du}{u}$ and $\sec(x)dx = \frac{-dv}{v}$

6. Trig Substitutions.

   For $\sqrt{a^2 - x^2}$ use $x = a\sin(\theta)$
   For $\sqrt{a^2 + x^2}$ use $x = a\tan(\theta)$
   For $\sqrt{x^2 - a^2}$ use $x = a\sec(\theta)$

7. Center of Mass.

   $\bar{x} = \int_a^b [f(x) - g(x)]x\,dx/A$
   $\bar{y} = \int_a^b [f(x)]^2 - [g(x)]^2\,dx/(2A)$
   where $A = \int_a^b [f(x) - g(x)]\,dx$.