Q1]...[10 points] Use the method of Lagrange Multipliers to find the shortest distance from the point $(1, 2, 3)$ to the curve of intersection of the surface $z = 1 - x^2 - y^2$ and the plane $x + y - z = 0$. Write down the function that is to be extremized, and write down the Lagrange Multiplier equations (how many equations? how many unknowns?). You do **not** have to solve these equations.

Q2]...[10 points] Find the direction in which the function $f(x, y, z) = x^2 + 2xy + y^3 - 3xz$ has the maximum rate of change at the point $(1, 0, 2)$. Also, find the directions in which the rate of change of f is zero at the point $(1, 0, 2)$.

Q3]. . . [10 points] You are given that $z = f(x, y)$, $x = x(t)$ and $y = y(t)$, where all functions have continuous second (partial) derivatives.
Write $\frac{dz}{dt}$ in terms of partials of z with respect to x and y and derivatives of x and y with respect to t.

Write $\frac{d^2z}{dt^2}$ in terms of partials of z with respect to x and y and derivatives of x and y with respect to t.

Q4]. . . [10 points] Write down (**do not evaluate**) multiple integral expressions for the volume of the region which is inside the sphere $x^2 + y^2 + z^2 = a^2$ and above the cone $z^2 = x^2 + y^2$, in

(a) Spherical Coordinates

(b) Cylindrical Coordinates

Q5]... [10 points] Find a double integral expression for the surface area of the portion of the surface $z = 4 - x^2 - y^2$ which lies above the plane $z = 3$. Write your double integral in *Polar Coordinates*. You do **not** have to evaluate your integral.

Q6]... [10 points] Compute the flux of the vector field $\mathbf{F} = \langle y, -x, 4 \rangle$ upward (that is use an upward pointing normal) through the portion of the surface $z = 1 - x^2 - y^2$ which lies in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$).

Q7]. . . [10 points] Let $\mathbf{F} = \langle x + y, x - z, z - y \rangle$.

- Compute curl($\mathbf{F}$).

- Find a function ϕ so that $\nabla \phi = \mathbf{F}$.

- Evaluate (by whatever way you can) $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the straight line path from $(1, 0, -1)$ to $(0, -2, 3)$.

Q8]... [**10 points**] Use Green's Theorem to compute the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (you'll end up evaluating a suitable line integral).

Q9]. . . [10 points] Use Stokes Theorem to evaluate

$$\oint_C y dx + z dy + x dz$$

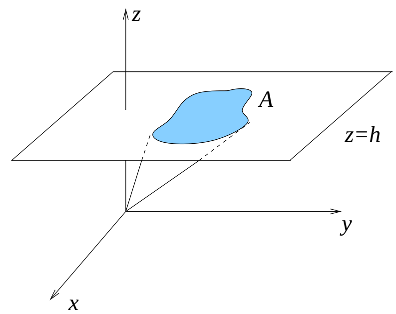
where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = 4$ with the plane $x + y + z = 0$, oriented so as to be consistent with the normal vector of the plane which points into the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$).

Q10]. . . [10 points] Use the Divergence Theorem to evaluate

$$\iint_S (x + y^2 + z^3) dS$$

where S is the unit sphere centered on the origin.
[Hint. First write $(x + y^2 + z^3)dS$ as $\mathbf{F} \cdot d\mathbf{S}$ for suitable vector field $\mathbf{F}$.]

Bonus]... Use the Divergence Theorem to prove the volume of the cone with vertex at the origin, and base of area A, bounded by a piecewise smooth, simple closed curve C on the plane $z = h$ is given by the usual formula $\frac{Ah}{3}$.



[Hint. Think about the vector field $\mathbf{F} = \langle x, y, z \rangle$]

Rough Work Page