Calculus IV [2443–004] Midterm II

Friday, March 12, 1999

For full credit, give reasons for all your answers.

Q1]...[15 points] For the double integral below, first sketch the region of integration, and then convert it to a polar coordinares integral.

$$\int_0^2 \int_{-\sqrt{2y-y^2}}^{\sqrt{2y-y^2}} f(x,y)\,dxdy$$

Q2]...[15 points] Use double integrals to find the volume of the region which is under the paraboloid $z = x^2 + y^2$ and above the region bounded by $x = y^2$ and $y = x - 6$.

Q3]...[15 points] Find the surface area of the part of the paraboloid

$$z = 4 - x^2 - y^2$$

which lies above the plane $z = 2$ and is contained in the first octant [ie. $x \geq 0$, $y \geq 0$, and $z \geq 0$].

Q4]...[15 points] Use the method of Lagrange Multipliers to find the maximum and minimum values of the function $f(x,y,z) = xyz$ on the sphere $x^2 + y^2 + z^2 = 12$.