Q1

$$f(x) = \sqrt{x}\, \tan(x)$$

$$f'(x) = \frac{d}{dx}\left(\sqrt{x}\, \tan(x)\right)$$

$$= \frac{d\sqrt{x}}{dx}\tan(x) + \sqrt{x}\,\frac{d \tan x}{dx} \quad \text{--- Product Rule}$$

$$= \frac{1}{2}x^{-\frac{1}{2}}\tan(x) + \sqrt{x}\,\sec^2(x) \quad \text{---- Power Rule + Trig derivatives}$$

$$y = \sin^4\left(\frac{x}{2x-1}\right)$$

$$\frac{dy}{dx} = \frac{d}{du}(u^4)\,\frac{du}{dx} \quad \text{--- ch. Rule} \qquad u = \sin\left(\frac{x}{2x-1}\right)$$

$$= 4u^3\,\frac{d \sin v}{dv}\,\frac{dv}{dx} \quad \text{--- ch. Rule Again} \qquad v = \frac{x}{2x-1}, \quad u = \sin(v)$$

$$= 4u^3 \cos(v)\,\frac{\frac{dx}{dx}(2x-1) - x\frac{d(2x-1)}{dx}}{(2x-1)^2}$$

$$= 4\sin^3\left(\frac{x}{2x-1}\right)\cos\left(\frac{x}{2x-1}\right)\frac{1(2x-1) - x(2)}{(2x-1)^2}$$

$$x^{2013} \to 2013\,x^{2012} \to (2013)(2012)\,x^{2011} \to \cdots \to (2013)\ldots(3)(2)(1)$$

$$\cos(2x-1) \to -(2)\sin(2x-1) \to -(2)^2\cos(2x-1) \to \cdots \to (2)^{2013}(??)$$

PTO.

$$\begin{tikzcd} \cos \arrow[r, "1"] & -\sin \arrow[r, "2"] & -\cos \arrow[r, "3"] & \sin \arrow[lll, bend left, "4"] \end{tikzcd}$$

$4\,\underline{)2013}$
$503 \quad R\,1$

$2013 = 4(503)$ & Remainder 1

so same effect on trig as 1st deriv

namely $\underline{\underline{-\sin}}$

$$g^{(2013)}(x) = (2013)(2012)\ldots(3)(2)(1) - (2)^{2013}\sin(2x-9)$$

Q2

$$\lim_{x\to 2} \frac{(x+3)^2 - 25}{x-2} = f'(2), \ldots \text{ where } f(x) = (x+3)^2$$

$$f'(x) = 2(x+3)$$

$$= 10 \qquad f'(2) = 2(5) = 10$$

$$\lim_{x\to\infty} \frac{4 - 3x - x^3}{(2x-1)^3} = \lim_{x\to\infty}\left(\frac{\frac{4}{x^3} - \frac{3}{x^2} - 1}{\left(2 - \frac{1}{x}\right)^3}\right)$$

$$= \frac{0 - 0 - 1}{(2-0)^3} = \boxed{-\frac{1}{8}}$$

$$\lim_{\theta\to 0}\left(\frac{\sin(3\theta)}{3\theta} \cdot 3\,\frac{1}{\frac{\theta + \tan\theta}{\theta}}\right) = 1 \cdot 3 \cdot \frac{1}{1+1} = \boxed{\frac{3}{2}}$$

Note we're using $\lim_{x \to 0} \left(\frac{\sin(x)}{x}\right) = 1$ twice here

once as $\frac{\sin(3\theta)}{3\theta} \longrightarrow 1$

and once as $\frac{\tan\theta}{\theta} = \frac{\sin\theta}{\theta} \cdot \frac{1}{\cos\theta} \to 1 \cdot \frac{1}{1} = 1$

Q3 $\int \frac{x}{\sqrt{x^2-1}}\, dx$

let $u = x^2 - 1$

$\frac{du}{dx} = 2x \qquad \frac{du}{2} = x\,dx$

$$\int = \int \frac{du}{2\sqrt{u}} = \frac{1}{2}\int u^{-\frac{1}{2}}\, du$$

$$= \frac{1}{2}\left(\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) + C$$

$$= \sqrt{u} + C$$

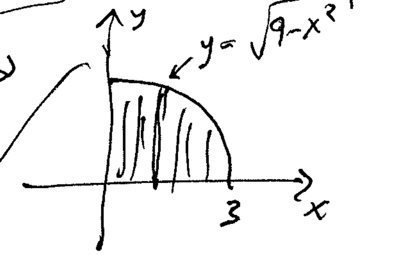
$$= \sqrt{x^2-1} + C$$

$$\int_3^0 \sqrt{9-x^2} + x^2 - 2\, dx = \int_0^3 2 - x^2 - \sqrt{9-x^2}\, dx$$

change limits ⇒ change sign of integrand.

$$= \left[2x - \frac{x^3}{3}\right]_0^3 - \underbrace{\int_0^3 \sqrt{9-x^2}\,dx}$$

$x^2+y^2=9$
circle



Area of $\frac{1}{4}$ circle of rad. 3

$$= \frac{1}{4}\pi(3)^2$$

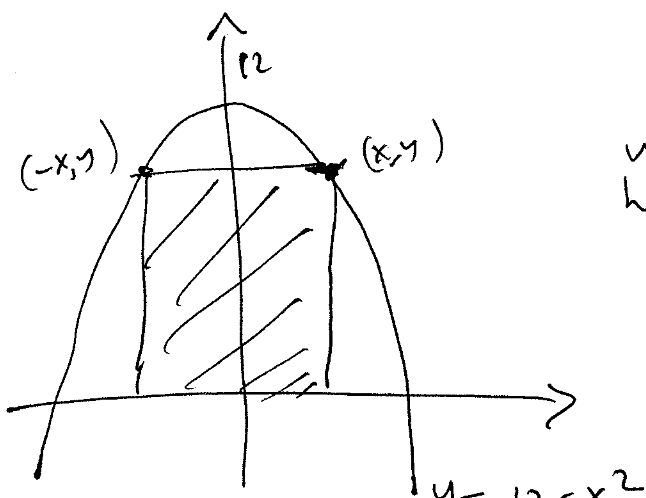
$$= \frac{9\pi}{4}$$

(geometry does this part!)

$$= (6-9) - \frac{9\pi}{4}$$

$$= -3 - \frac{9\pi}{4}$$

Q4



width $= 2x$

height $= y = 12 - x^2$

$$\text{Area} = (2x)(12-x^2)$$

$$= 24x - 2x^3$$

$$\frac{dA}{dx} = 24 - 6x^2$$

$$\frac{dA}{dx} = 0 \qquad 6x^2 = 24$$

$$x^2 = 4$$

$$x = \pm 2$$

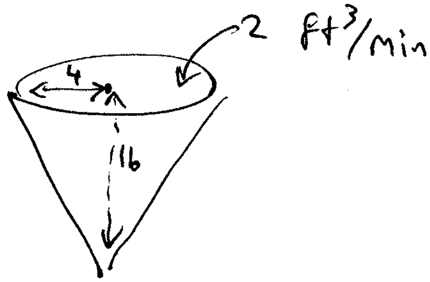
Ans

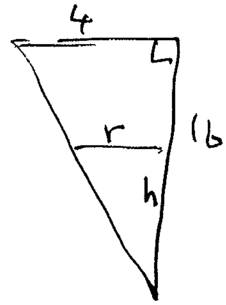
height $= 12 - (2)^2 = 8$

width $= 2(2) = 4$

Area $= 32$

Q5





Similar Δ's $\Rightarrow \dfrac{h}{16} = \dfrac{r}{4} \Rightarrow r = \dfrac{h}{4}$

$$\text{Vol} = \frac{1}{3}\pi r^2 h \quad \text{--- (vol of a cone!)}$$

$$= \frac{1}{3}\pi \left(\frac{h}{4}\right)^2 h$$

$$= \frac{\pi}{48} h^3$$

$$\frac{dV}{dh} = \frac{3\pi}{48} h^2 = \frac{\pi h^2}{16}$$

$$\underset{\substack{\uparrow \\ \text{TOLD}}}{2} = \frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} = \frac{\pi h^2}{16}\underset{\substack{\uparrow \\ \text{want this} \\ \text{when } h = 5}}{\frac{dh}{dt}}$$

$$2 = \frac{\pi (5)^2}{16}\frac{dh}{dt} \Rightarrow \boxed{\frac{dh}{dt} = \frac{32}{\pi(25)} \text{ ft/min}}$$

Q6

$$x^2y - y^3 = 8$$

Implicit Diff $\Rightarrow$

$$\frac{d}{dx}\left(x^2y - y^3\right) = \frac{d8}{dx}$$

Prod. Rule   Ch. Rule

$$2xy + x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$$

$$2xy = \left(3y^2 - x^2\right)\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2xy}{3y^2 - x^2}$$

At $(3,-1)$ ... $\frac{dy}{dx} = \frac{2(3)(-1)}{3(-1)^2 - (3)^2} = \frac{-6}{-6}$

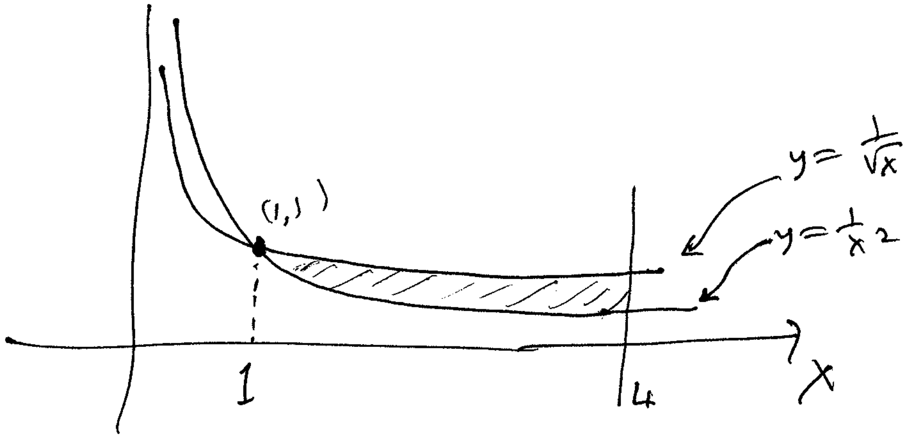
$$= 1.$$

$\boxed{\text{slope} = 1}$

$(y - (-1)) = 1(x - 3)$ $\leftarrow$ slope, point formula for Line.

$\boxed{y + 1 = x - 3}$ Tangent line

Q7



$$\text{Area} = \int_1^4 \frac{1}{\sqrt{x}} - \frac{1}{x^2}\,dx$$

$$= \int_1^4 \left(x^{-\frac{1}{2}} - x^{-2}\right) dx$$

$$= \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{-1}}{-1}\right]_1^4$$

$$= \left[2\sqrt{x} + \frac{1}{x}\right]_1^4$$

$$= \left(2\sqrt{4} + \frac{1}{4}\right) - \left(2\sqrt{1} + 1\right)$$

$$= 4 + \frac{1}{4} - 3 = 1 + \frac{1}{4} = \frac{5}{4}$$

Q8 $L(x)$ for $f(x) = \sqrt[3]{x}$ at 64 typo!

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$f(64) = \sqrt[3]{64}$
$= 4$

$f'(64) = \frac{1}{3(\sqrt[3]{64})^2}$

$\sqrt[3]{64} = 4$

$= \frac{1}{3(4)^2}$

$= \frac{1}{48}$

$L(x) = f(64) + f'(64)(x-64)$

$$\boxed{L(x) = 4 + \frac{1}{48}(x-64)}$$

$\sqrt[3]{64.048} = f(64.048)$

$\approx L(64.048)$

$= 4 + \frac{1}{48}(64.048 - 64)$

$= 4 + \frac{1}{48}(0.048)$

$= 4 + 0.001 = \underline{4.001}$

Q9

$g(x) \Downarrow$ where $g'(x) < 0$

ie. on $(-\infty, -6)$

& $(0, 3)$

& $(7, \infty)$

$g(x) \Uparrow$ where $g'(x) > 0$

ie. on $(-6, 0)$ and $(3, 7)$

Extrema where $g'(x) = 0$

ie. -6, 0, 3, and 7

We know $g(x) \Downarrow$ on $(-\infty, -6)$ & $g(x) \Uparrow$ on $(-6, 0)$

$\Rightarrow$ local min at -6

Likewise Local min at 3

& local max at 0

& local max at 7

Q10 (Riemann sum version of 1st part)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{7i^2}{n^3} + \frac{5}{n} \right)$$

n^3 (not n^2 --- typo)

The one on the exam is NOT a Riemann sum — limit works out to be ∞.

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(7\left(\frac{i}{n}\right)^2 + 5 \right)\left(\frac{1}{n}\right)$$

interval $= \frac{1-0}{n} \Rightarrow [0,1]$

$f(x) = 7x^2 + 5$

Limit of Riemann sums.

$$= \int_0^1 (7x^2+5)\,dx = \left[\frac{7x^3}{3} + 5x\right]_0^1 = \frac{7}{3} + 5$$

$$L_n = \sum_{i=1}^{n} 3\sqrt{7 + \frac{4(i-1)}{n}} \cdot \frac{4}{n}$$

$f(x) = 3\sqrt{7+x}$

interval $= [0,4]$ say.

$$\int_0^4 3\sqrt{7+x}\,dx$$

Q10 1st part (as written)

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{7\, i^2}{n^2} + \frac{5}{n} \right)$$

$$= \lim_{n\to\infty} \left[\frac{7\left(\sum_{i=1}^{n} i^2\right)}{n^2} + \frac{5}{n}\left(\sum_{i=1}^{n} 1\right) \right]$$

$$= \lim_{n\to\infty} \left[\frac{7\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right)}{n^{\cancel{2}}} + \frac{5}{n}(n) \right]$$

$$= \lim_{n\to\infty} \left(\frac{7}{6}\, \frac{(n+1)(2n+1)}{n} + 5 \right)$$

$$= \infty + 5$$

$$= \infty$$

power of n above line $= 2$
power of n below $= 1$
$\Rightarrow \to \infty$