Calculus IV [2443–004] Midterm I
Friday, February 12, 1999
For full credit, give reasons for all your answers.

Q1]...[15 points] Draw the level curves $f = 0$, $f = 1$, $f = 4$, and $f = -1$ for the function $f(x, y)$ below. Also, sketch the graph of f in a neighborhood of the origin.

$$f(x, y) = x^2 - y^2$$

Does the following limit exist? Give reasons for your answer.

$$\lim_{(x,y) \to (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

Q2]...[10 points] If $x = t \sin s$ and $y = t \cos s$ find

$$\frac{\partial^2}{\partial s \partial t} f(x, y)$$

Q3]...[10 points] Write down the differential for the function

$$f(x, y) = y e^{x + y^2}$$

Use the differential above to estimate the value of $f(-3.92, 2.05)$.

Q4]...[15 points] Compute the gradient ∇f for the function

$$f(x, y, z) = y e^{-x^2} \sin z$$

Find the direction in which f is increasing most rapidly at the point $(0, 1, \pi/3)$.
What is the maximum rate of change of f at the point $(0, 1, \pi/3)$?
Find an equation for the tangent plane to the level surface of f at the point $(0, 1, \pi/3)$.

Q5]...[10 points] Find and classify the critical points of the function

$$f(x, y) = 2x^3 - 6xy - 3y^2$$