

MATH 2934 – Additional problem assigned on 7/17/14

Additional Problem.

Let \mathcal{L} stand for the set of all linear functions of one variable, i.e., \mathcal{L} consists of all functions of the form $f(x) = ax + b$ (where a and b are constants). Clearly, the quadratic function $q(x) = x^2$ does not belong to \mathcal{L} . Suppose that you would like to find a linear function f from \mathcal{L} that is the best approximation of q on the interval $[0, 1]$, in the sense that f is such that the “error” defined by the integral

$$\int_0^1 [q(x) - f(x)]^2 dx = \int_0^1 [q(x) - (ax + b)]^2 dx = \frac{a^2}{3} + b^2 + ab - \frac{a}{2} - \frac{2b}{3} + \frac{1}{5}.$$

- (a) Find the values of a and b for which the “error” is the smallest.

Hint: The answer is $a = 1$, $b = -\frac{1}{6}$, but you have to show me how you can obtain these values.

- (b) Based on your answer in part (a) (which was actually given in the hint), write down the function f from \mathcal{L} that is “closest” to the function q , where by “distance” between f and q we mean the “error” defined in part (a).
- (c) What is the value of the “error” in approximating the function $q(x)$ with the function obtained in part (b)?

Remark: This problem is similar to the problem about the *method of least squares* in Exercise 14.7/55 of the book.