H.w. #6

3.2 The Derivative As a Function

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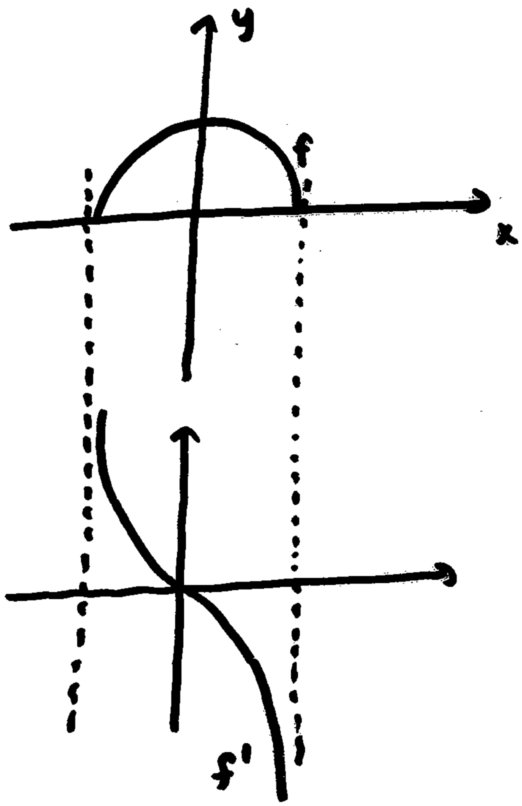
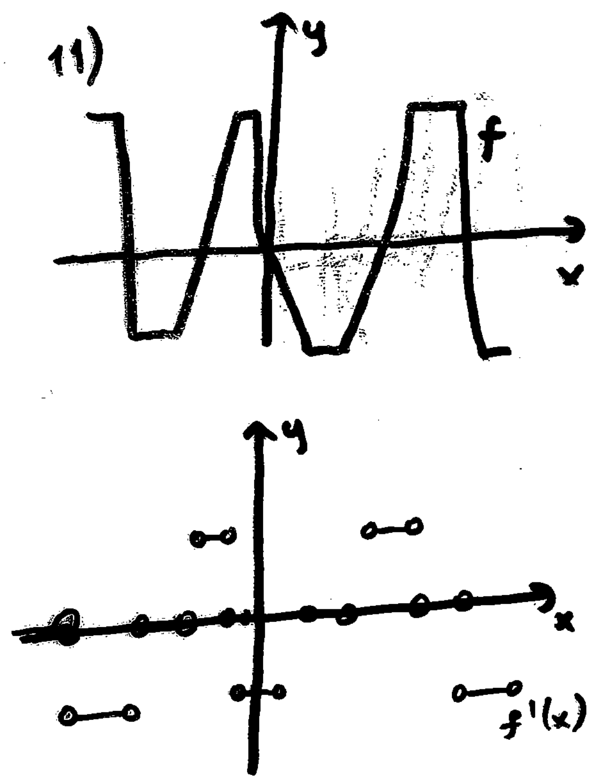
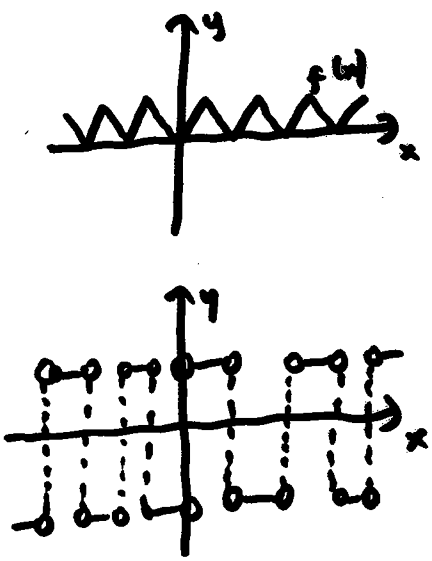
10)



11)



12)



20) $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} \dfrac{[12 + 7(x+h)] - (12 + 7x)}{h}$

$= \lim\limits_{h \to 0} \dfrac{12 + 7x + 7h - 12 - 7x}{h} = \lim\limits_{h \to 0} \dfrac{7h}{7} = 7$

Domain(f)= Domain $(f')=\mathbb{R}$

24) $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} \dfrac{(x+h+\sqrt{x+h}) - x + \sqrt{x}}{h}$

$= \lim\limits_{h \to 0} \dfrac{h}{h} + \dfrac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \dfrac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$

$= \lim\limits_{h \to 0} \left(1 + \dfrac{x+h-x}{h.(\sqrt{x+h} + \sqrt{x})}\right) = \lim\limits_{h \to 0} \left(1 + \dfrac{1}{\sqrt{x+h} + \sqrt{x}}\right)$

$= 1 + \dfrac{1}{\sqrt{x} + \sqrt{x}} = 1 + \dfrac{1}{2\sqrt{x}}$

28) $g'(x) = \lim\limits_{h \to 0} \dfrac{g(x+h) - g(x)}{h} = \lim\limits_{h \to 0} \dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

$= \lim\limits_{h \to 0} \dfrac{x^2 - (x+h)^2}{h\,(x+h)^2 x^2} = \lim\limits_{h \to 0} \dfrac{x^2 - x^2 - 2xh - h^2}{h.(x+h)^2.x^2}$

$= \lim\limits_{h \to 0} \dfrac{-2xh - h^2}{h.(x+h)^2.x^2} = \lim\limits_{h \to 0} \dfrac{\cancel{h}(-2x-h)}{\cancel{h}.(x+h)^2.x^2} = \dfrac{-2x}{x^2.x^2} = \dfrac{-2}{x^3}$ //

36)

a) g is discontinuous at; $x=-2$ (removable discontinuity)

$x=0$ (g is not defined)

$x=5$ (Jump discontinuity)

b) g is not differentiable at the points $x=-2$, $x=0$ and $x=5$ since g is discontinuous.

g is also not differentiable at; $x=-1$ (corner)

$x=2$ (vertical tang

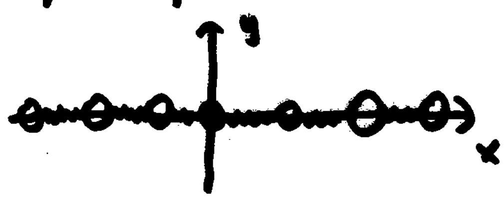
$x=4$ (" "

39) a) $f'(a)=\dfrac{1}{3a^{2/3}}$

b) Limit does not exist at 0 hence $f'(0)$ DNE.

c) $\lim\limits_{x\to 0}|f'(x)| = \lim\limits_{x\to 0}\dfrac{1}{3x^{2/3}} = \infty$ and f is continuous at $x=0$ (root function), so f has a vertical tangent at $x=0$.

42) $f(x)=[\![x]\!]$ is not continuous at any integer n, so f is not differentiable at any n. If a is not an integer, then f is constant on an open interval containing a, so $f'(a)=0$. Thus, $f'(x)=0$, x is not an intege



3.3 - Differentiation Formulas

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6) $g(x) = 5x^8 - 2.x^5 + 6 \Rightarrow g'(x) = 5.8.x^7 - 2.5.x^4 + 0$

$= 40x^7 - 10x^4$

10) $R(t) = 5t^{-3/5} \Rightarrow R'(t) = 5.\left(\frac{-3}{5}\right).t^{-3/5-1} = -3.t^{-8/5}$

14) $f(t) = \sqrt{t} - \frac{1}{\sqrt{t}} = t^{1/2} - t^{-1/2} \Rightarrow f'(t) = \frac{1}{2}.t^{1/2-1} - \left(\frac{-1}{2}\right)t^{-\frac{1}{2}-1}$

$= \frac{1}{2}t^{-1/2} + \frac{1}{2}t^{-\frac{3}{2}}$

28) $f(t) = \frac{2t}{4+t^2} \Rightarrow f'(t) = \frac{2.(4+t^2) - 2t.(2t)}{(4+t^2)^2}$

$= \frac{8 + 2t^2 - 4t^2}{(4+t^2)^2} = \frac{8 - 2t^2}{(4+t^2)^2}$

32) $y = \frac{\sqrt{x}-1}{\sqrt{x}+1} \Rightarrow y' = \frac{\left(\frac{1}{2\sqrt{x}}\right).(\sqrt{x}+1) - \left(\frac{1}{2\sqrt{x}}\right).(\sqrt{x}-1)}{(\sqrt{x}+1)^2}$

$= \frac{\frac{1}{2} + \frac{1}{2\sqrt{x}} - \frac{1}{2} + \frac{1}{2\sqrt{x}}}{(\sqrt{x}+1)^2}$

$= \frac{1}{\sqrt{x}.(\sqrt{x}+1)^2}$

34) $y = x^2 + x + x^{-1} + x^{-2} \Rightarrow y' = 2x + 1 - x^{-2} - 2x^{-3}$

58) Given : $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$ and $g'(3) = 5$

a) $(f+g)'(3) = f'(3) + g'(3) = -6 + 5 = -1$

b) $(f \cdot g)'(3) = f(3) . g'(3) + f'(3) . g(3) = 4.5 + (2).(-6)$

$= 20 - 12 = 8$

c) $\left(\frac{f}{g}\right)'(3) = \dfrac{g(3).f'(3) - f(3).g'(3)}{[g(3)]^2} = \dfrac{(+2)(-6) - 4.5}{2^2}$

$= -8$

d) $\left(\frac{f}{f-g}\right)'(3) = \dfrac{[f(3) - g(3)].f'(3) - f(3)[f'(3) - g'(3)]}{[f(3) - g(3)]^2}$

$= \dfrac{(4-2).(-6) - 4(-6-5)}{(4-2)^2} = 8$

60) $\dfrac{d}{dx}\left[\dfrac{h(x)}{x}\right] = \dfrac{x.h'(x) - h(x).1}{x^2} \Rightarrow \dfrac{d}{dx}\left[\dfrac{h(x)}{x}\right]_2 = \dfrac{2.h'(2) - h(2)}{2^2}$

$= \dfrac{2.(-3) - 4}{4}$

$= -2.5$

64) a) $y = x^2 . f(x) \Rightarrow y' = 2x . f(x) + x^2 . f'(x)$

b) $y = \dfrac{f(x)}{x^2} \Rightarrow y' = \dfrac{x^2 . f'(x) - f(x) . (2x)}{(x^2)^2}$

c) $y = \dfrac{x^2}{f(x)} \Rightarrow y' = \dfrac{(2x) . f(x) - x^2 . f'(x)}{[f(x)]^2}$

d) $y = \dfrac{1 + x . f(x)}{\sqrt{x}} \Rightarrow y' = \dfrac{(1 + x . f(x))' . \sqrt{x} - \frac{1}{2\sqrt{x}} . (1 + x . f(x))}{(\sqrt{x})^2}$

$= \dfrac{(1 . f(x) + x . f'(x)) . \sqrt{x} - \frac{1}{2\sqrt{x}} (1 + x . f}{x}$

68) $y = \dfrac{x-1}{x+1} \Rightarrow y' = \dfrac{(x+1) . 1 - (x-1) . 1}{(x+1)^2} = \dfrac{2}{(x+1)^2}$

If the tangent intersects the curve when $x = a$ then its slope is $\dfrac{2}{(a+1)^2}$. But if the tangent is parallel to $x - 2y = 2$ that is, $y = \frac{1}{2}x - 1$, then its slope is $\frac{1}{2}$.

Thus, $\dfrac{2}{(a+1)^2} = \dfrac{1}{2} \Rightarrow (a+1)^2 = 4 \Rightarrow a + 1 = \mp 2$

$\Rightarrow a = 1$ or $a = -3$

when $a = 1$, tangent line equation is $y - 0 = \frac{1}{2}(x - 1)$

when $a = -2$, $y = 2$, " " " $y - 2 = \frac{1}{2}(x + 3)$

73) a) $(f\cdot g\cdot h)' = [(f\cdot g)\cdot h]' = (f\cdot g)'\cdot h + (f\cdot g)\cdot h'$

$= (f'\cdot g + f\cdot g')h + f\cdot g\cdot h'$

$= f'\cdot g\cdot h + f\cdot g'\cdot h + f\cdot g\cdot h'$

b) $y = \sqrt{x}\,(x^4+x+1)(2x-3)$, by using part a,

$y' = \frac{1}{2\sqrt{x}}(x^4+x+1).(2x-3) + \sqrt{x}\,(4x^3+1).(2x-3)$

$+ \sqrt{x}\,(x^4+x+1).\,2.$

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74) a) Putting $f=g=h$ in part a of 73

$\frac{d}{dx}[f(x)]^3 = (f f f)' = f'\cdot f\, f + f\cdot f'\cdot f + f\cdot f\cdot f'$

$= 3.[f(x)]^2.\, f'(x)$

b) $y = (x^4+3x^3+17x+82)^3$

$\Rightarrow y' = 3.(x^4+3x^3+17x+82)^2.(4x^3+9x^2+17)$