

Monday 09/26/2005

Midterm I

8:30am–9:20am

Name: Student ID: **Instructions.**

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	10	
Q2	10	
Q3	10	
Q4	10	
Q5	10	
TOTAL	50	

Q1]... [10 points] Complete the truth table for the proposition below. The extra columns are there for you to use to compute truth values of intermediate expressions (if you wish).

P	Q	R				$\neg(\neg R \rightarrow Q) \vee (P \wedge \neg R)$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Q2]. . . [10 points] Find a **disjunctive normal form** expression (involving \wedge , \vee , \neg , and P , Q , R) which has the following truth table. Show the steps of your work.

P	Q	R	
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

Find a **conjunctive normal form** expression (involving \wedge , \vee , \neg , and P , Q , R) which has the same truth table above. Show the steps of your work.

Q3]. . . [10 points] Let $P(x, y)$ be the statement “ x and y satisfy the equation $2x + y = 4$ ”. Determine which of the following are true; the universe for x and y is the set of real numbers. Justify your answers.

1. $\forall x \exists y P(x, y)$

2. $\forall x \forall y P(x, y)$

3. $\exists x \exists y P(x, y)$

4. $\exists x \forall y P(x, y)$

Let the universe of x be all the people in the world, let $F(x)$ denote “ x is friendly”, let $T(x)$ denote “ x is tall”, and $A(x)$ denote “ x is angry”. Translate the following statements into predicate statements with suitable quantifiers.

1. All tall people are friendly.

2. Some tall people are friendly.

3. No friendly people are angry.

4. There is precisely one tall, angry person.

Q4]... [10 points] Give a **direct proof** of the following. *If m and n are odd integers, then their product is also odd.*

Prove the following **by contradiction**. *If n is an integer and $3n^2 + 8$ is even, then n is also even.*

Q5]... [10 points] Suppose $A = \{a, b, c\}$. Say whether the following are true or false.

1. $\{a\} \in A$.
2. $\{a\} \in \mathcal{P}(A)$.
3. $b \in A$.
4. $\emptyset \in \mathcal{P}(A)$.
5. $\{\emptyset\} \subset \mathcal{P}(A)$.
6. $\{a, b\} \in \mathcal{P}(A)$.
7. $(a, c) \in A \times A$.
8. $|A \times A| = 2^3$.
9. $|\mathcal{P}(A)| = 3^2$.
10. $|\{(x, y) \in A \times A \mid x \neq y\}| = 6$.