Thursday 12/15/2005   Final Examination   1:30pm–3:30pm

Name:

Student ID:

Instructions.

1. Attempt all questions.

2. Do not write on back of exam sheets. Extra paper is available if you need it.

3. Show all the steps of your work clearly.

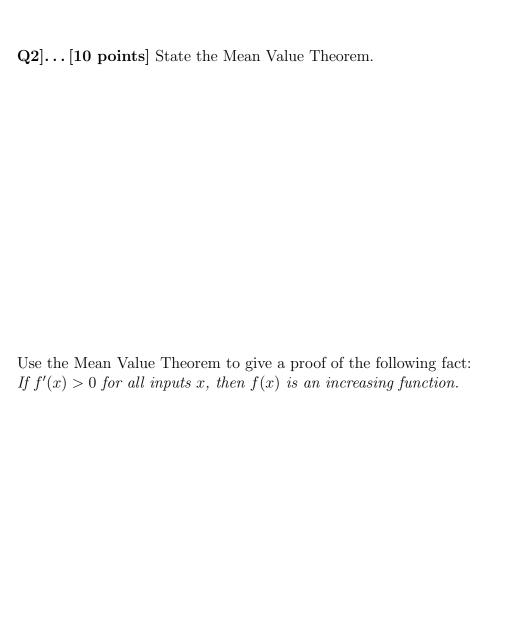| Question	Points	Your Score
Q1	15	
Q2	10	
Q3	10	
Q4	10	
Q5	10	
Q6	15	
Q7	15	
Q8	15	
TOTAL	100	

Q1]... [15 points] Write down the limit definition of the derivative of the function $f(x)$ at the point a.

Use the limit definition to compute the derivative of the function $f(x) = x^2$. [Do not use power rule!]

Compute the following limit

$$\lim_{x \to \pi/6} \frac{\cos(2x) - (1/2)}{x - (\pi/6)}$$

Hint: Think about limit definitions of derivatives!

Q2].. . [10 points] State the Mean Value Theorem.

Use the Mean Value Theorem to give a proof of the following fact:
If $f'(x) > 0$ for all inputs x, then $f(x)$ is an increasing function.

Q3]... [10 points] Compute the following higher derivatives.

1. $f^{(100)}(x)$ where $f(x) = 19x^{54} + 2x^{25} - 13x + 4$.

2. $f^{(137)}(x)$ where $f(x) = \sin(3x + 2)$.

Q4]...[10 points] Find the equation of the tangent line to the implicit curve

$$x + 2y + 1 = \frac{y^2}{x-1}$$

at the point $(2, -1)$.

Q5].... [10 points] A police officer standing near a highway is using a radar gun to catch speeders. The officer aims the radar gun at a car that has just passed by and, when the radar gun is pointing at an angle of $\pi/4$ to the direction of the highway, the officer notes that the distance between the car and the radar gun is increasing at a rate of 100km/hr. Compute the speed of the car at that instant?

Q6].. . [15 points] Find the maximum volume of a right-circular cone which can be inscribed inside of a sphere of radius R. Recall that the volume of a right-circular cone is $(1/3)\pi r^2 h$ where r is the radius of the base of the cone, and h is the height of the cone.

Q7]. . . [15 points] Consider the function

$$f(x) = \frac{x^2}{x^2 + 1}$$

Answer the following questions about $f(x)$ and **draw the graph** of $f(x)$.

1. Determine the x- and y-intercepts of f.

2. Does the graph of $f(x)$ have any symmetry?

3. Does the graph of $f(x)$ have any vertical asymptotes?

4. Does the graph of $f(x)$ have any horizontal asymptotes?

5. Compute $f'(x)$.

6. Find the critical points, intervals where f is increasing/decreasing and local max/min for f.

7. Compute $f''(x)$.

8. Find intervals where f is concave up and where f is concave down, and inflection points.

Sketch the graph of $f(x)$ indicating the information above.

Q8]...[15 points] Compute the derivatives $f'(x)$ of the following three functions.

1. $f(x) = \sin^2(3x^3 - 4)$

2. $f(x) = (\sqrt{x-1})\tan(x)$

3. $f(x) = \frac{(x^2+2)}{(x-2)^2}$