Thursday 09/24/2009 | Midterm I | 9:00am–10:15am

Name: ____________ Student ID: ____________

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	12	
Q2	11	
Q3	12	
Q4	15	
Q5	15	
Q6	20	
Q7	15	
TOTAL	100	

Q1]. . . [12 points] Find a **disjunctive normal form** expression (involving $\wedge$, $\vee$, $\neg$, and P, Q, R) which has the following truth table. Show the steps of your work.

P	Q	R	
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	T

Find a **conjunctive normal form** expression (involving $\wedge$, $\vee$, $\neg$, and P, Q, R) which has the same truth table above. Show the steps of your work.

Q2]. . . [11 points] Write down the distributive law for $\wedge$ over $\vee$.

Write down the distributive law for $\vee$ over $\wedge$.

Write down the two De Morgan laws (involving negations of $\wedge$ and $\vee$ statements).

Use the De Morgan and distributive laws to show that the expression

$$[P \wedge (\neg Q) \wedge R] \ \vee \ [P \wedge (\neg Q) \wedge (\neg R)] \ \vee \ [P \wedge Q \wedge R] \ \vee \ \neg[(\neg P) \vee (\neg Q) \vee R]$$

is logically equivalent to P.

Q3]. . . [12 points] Are the following two expressions logically equivalent. If you say so, please explain why. If you say not, then please give an example which shows that they are different.

$$\forall x[P(x) \rightarrow Q(x)]$$

and

$$(\forall x P(x)) \rightarrow (\forall x Q(x))$$

Same question for the expressions

$$\exists x[P(x) \vee Q(x)]$$

and

$$(\exists x P(x)) \vee (\exists x Q(x))$$

Q4]. . . [15 points] Give a **direct proof** of the following. *If m and n are odd integers, then their product is also odd.*

Write down the contrapositive of the following statement about integers n. *If n^3 is even, then n is also even.*

Prove the statement *"If n^3 is even, then n is also even"* by giving a proof of its contrapositive.

Q5]. . . [15 points] Give a proof of the following: *The cube root of 2 is irrational.* You are free to cite the results of **Q4** if they are of any help to you.

Q6]. . . [20 points] State the principle of induction.

Give a proof by induction of the following. *For each positive integer n,*

$$1^2 + \cdots + n^2 \ = \ \frac{n(n+1)(2n+1)}{6}$$

Q7]. . . [15 points] Give a proof by induction of the following. $2^{2n-1} + 3^{2n-1}$ *is a multiple of* 5 *for all integers* $n \geq 1$.