Mid II Review questions on Permutations + Isometries

Q1. Find compositions (products) below in $\mathrm{Perm}(\{1,2,3,4,5\})$.

(i) $(243)(123)(45)$

(ii) $(12)(23)(34)(45)$

(iii) $(123)(2534)(132)$

(iv) $(12)(3142)(12)$

Q2. Find the compositions below in $\mathrm{Isom}(\mathbb{E}^2)$.

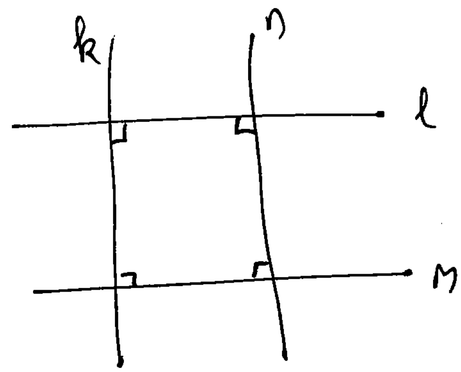
(i) $R_2 \circ R_1$ $\quad$ $R_1 = 180°$ rotation about $(0,0)$
$R_2 = 180°$ rotation about $(1,0)$

(ii) $R_1 \circ R_2$ $\quad$ same R_1, R_2 as above.

(iii) $R_4 \circ R_3$ $\quad$ $R_3 = 90°$ counterclockwise rot^{n} about $(0,0)$
$R_4 = $ - - - - - - - - $(1,0)$

(iv) $R_4 \circ R_3^{-1}$

Q3. Find composites



$k\,n\,l\,m$

$k\,l\,n\,m$

$k\,l\,m\,n$

$l\,k\,m\,n$

$l\,m\,k\,n$.

Q4. Find composite

$n\,m\,l$

l = reflection in x-axis

m = reflection in y-axis

n = reflection in line $(y=x)$

Q5 $\quad P_n$ = regular polygon with n sides in $\mathbb{E}^2$.

Label vertices of P_n by $1, 2, \ldots, n$.

In class notes $\quad \mathrm{Symm}(P_n)$ consists of those isometries of $\mathbb{E}^2$ which take the subset $P_n \subseteq \mathbb{E}^2$ to itself.

$\mathrm{Symm}(P_n)$ has no glides or translations. Just n rotations + n translations.

Given $f \in \mathrm{Symm}(P_n)$

we have $\quad f: \mathbb{E}^2 \longrightarrow \mathbb{E}^2 \quad$ preserving P_n as a set.

f also takes vertices of P_n to vertices of P_n

so we may restrict domain + codomain of f to get a bijection

$$f|_{\{1,\ldots,n\}} : \{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$$

This gives a map

$$\mathrm{Symm}(P_n) \longrightarrow \mathrm{Perm}(\{1, \ldots, n\})$$

Write this out explicitly in the case of a triangle $(n=3)$ & a square $(n=4)$.
