Friday 10/19/2007   Midterm I   50 minutes

Name: Student ID:

Instructions.

1. Attempt all questions.

2. Do not write on back of exam sheets. Extra paper is available if you need it.

3. Show all the steps of your work clearly.

4. No calculators, no notes, no books.

Question	Points	Your Score
Q1	10	
Q2	20	
Q3	12	
Q4	8	
TOTAL	50	

Q1]...[10 points] Differentiate the following function

$$f(x) = \tan\left(\sqrt{x^2 + 4x + 1}\right)$$

Suppose $f''(x)$ exists at all points x of an interval I. If f vanishes at three distinct points of I, show that f'' must vanish at some point of I.

Q2]...[20 points] Sketch the graph of the function

$$f(x) = x^{1/3}(x - 4)$$

after you have answered the following questions. Make sure that your answers to these questions are visible/highlighted on your graph.

1. Find the intercepts of $y = f(x)$ and determine the behavior of $f(x)$ as $x \to \infty$ and as $x \to -\infty$.

2. Compute the derivative $f'(x)$, and find all the critical points of $f(x)$.

3. Determine the intervals where $f(s)$ is increasing, and where $f(x)$ is decreasing.

4. Compute $f''(x)$, and determine the intervals where $f(x)$ is CCU, and where $f(x)$ is CCD. Does the graph of $f(x)$ have inflection points?

Now sketch the graph $y = f(x)$:

Q3].... [12 points] Show that if it is possible to draw three normal lines from the point $(a, 0)$ to the parabola $x = y^2$, then a must be greater than 2.

One of the three normals above is the x-axis. Find the value of a for which the other two normals are perpendicular to each other.

Find the value(s) of a for with the other two normals intersect at an angle of $\pi/3$.

Q4]... [8 points] Find the absolute maximum and the absolute minimum of the function

$$f(x) = x + \frac{4}{x}$$

on the interval $[1, 6]$.