<u>Th$^{\underline{m}}$</u> (Euclid) There are infinitely many prime numbers.

<u>Proof</u> We argue by contradiction. Assume that there are only finitely many prime numbers. List them!

$$p_1, \text{----}, p_n .$$

Consider the integer $M = (p_1 \cdot \text{---} \cdot p_n) + 1$

Note $p_i \nmid M$ for all $1 \leq i \leq n$,

because there is a remainder of 1 on dividing M by p_i.

Therefore either M is prime (and is distinct from $\{p_1 \text{—} p_n\}$) or the prime factors of M are distinct from $\{p_1 \text{—}, p_n\}$. In either case, there are prime numbers distinct from $\{p_1, \text{—}, p_n\}$. This contradicts the assumption that $p_1 \text{—}, p_n$ was a list of all the primes. Therefore the assumption that there is a finite list of all primes is false.

Therefore, there are infinitely many prime numbers. $\boxed{///}$