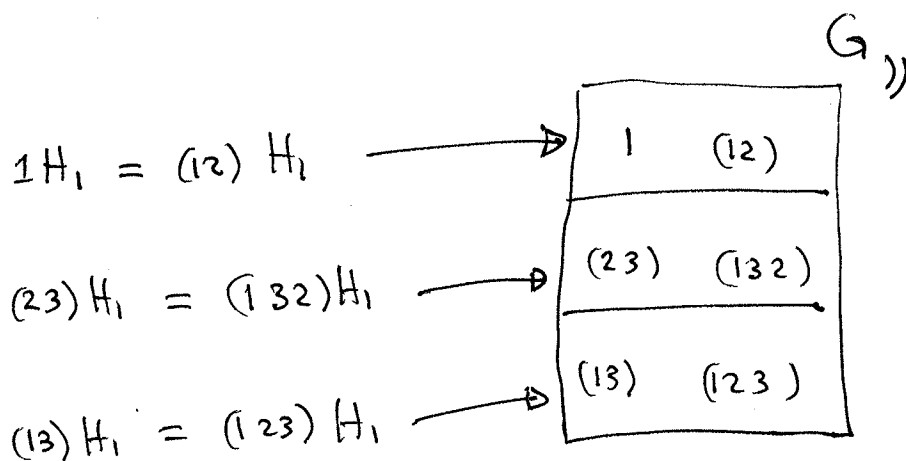


Explicit verification of Lagrange's Theorem proof for some subgroups of $\text{Perm}(\{1,2,3\})$. ①

$$G = \text{Perm}(\{1,2,3\}) = \{1, (12), (13), (23), (123), (132)\}$$

eg ① $H_1 = \{1, (12)\} < G$.



• $|L_g(H_1)| = |H_1|$ for each g (since L_g injective)

• The $L_g(H_1)$ sets are either equal or disjoint (empty intersection).

\Rightarrow they partition G into 3 disjoint sets each with $|H_1|$ elements.

$$\Rightarrow 3|H_1| = |G|$$

$$\Rightarrow |H_1| \mid |G|$$

eg 2

$$H_2 = \{1, (123), (132)\} < G.$$

$$\begin{array}{l}
 H_2 = (123)H_2 = (132)H_2 \longrightarrow \\
 (12)H_2 = (13)H_2 = (23)H_2 \longrightarrow
 \end{array}
 \begin{array}{|c|c|c|}
 \hline
 1 & (123) & (132) \\
 \hline
 (12) & (23) & (13) \\
 \hline
 \end{array}
 \begin{array}{l}
 \\
 \\
 G
 \end{array}$$

- $|L_g(H_2)| = |H_2|$ for each g (since L_g injective).
- The $L_g(H_2)$ sets are either equal or disjoint.

\Rightarrow they partition G into 2 disjoint sets, each with $|H_2|$ elements.

$$\Rightarrow 2|H_2| = |G|$$

$$\Rightarrow |H_2| \mid |G|$$

Remark $H_2 =$ subgroup, $\langle (12) \rangle$, generated by all powers of (12) .

$$\Rightarrow \text{order}((12)) = |H_2|, \text{ and so } \text{order}((12)) \mid |G|.$$

$H_2 =$ subgroup, $\langle (123) \rangle$, generated by all powers of (123) .

$$\Rightarrow \text{order}((123)) = |H_2|, \text{ and so } \text{order}((123)) \mid |G|.$$