Monday 09/08/2014 Midterm I 9:30–10:20am

Name: ____________ Student ID: ____________

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	25	
Q2	25	
Q3	25	
Q4	25	
TOTAL	100	

Q1]. . . [25 points]

1. Give the definition of an *odd integer*.

2. Give a detailed proof of the following proposition about integers n.

   If n is odd, then n^2 is odd.

3. Is the following proposition about integers n true or false? Why?

   If n^2 is even, then n is even.

Q2]. . . [25 points]

1. Write down the *converse* of the conditional statement $P \longrightarrow Q$.

2. Write down the *contrapositive* of the conditional statement $P \longrightarrow Q$.

3. Which of the two statements above are logically equivalent to the original statement $P \longrightarrow Q$?

4. For each of the following statements, say whether it is equivalent to the negation of a conditional: $\neg(P \longrightarrow Q)$. Give reasons for your answers.

   (a) $\neg P \vee Q$

   (b) $\neg P \wedge Q$

   (c) $P \wedge \neg Q$

   (d) $P \vee \neg Q$

Q3]... [25 points] Give a careful proof of the following proposition about real numbers x and y. If it helps, you may use the fact that the product of an arbitrary real number and 0 is equal to 0.

If $x \neq 0$ *and* $y \neq 0$*, then* $xy \neq 0$*.*

Q4]...[25 points] Let $P(x, y)$ be the predicate $x \leq y$. Say which of the following quantified statements are true for the universal set $\mathbb{N}$ of all positive integers. Give reasons to support your answers.

1. $(\forall x \in \mathbb{N})(\forall y \in \mathbb{N})P(x, y)$

2. $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})P(x, y)$

3. $(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})P(x, y)$

4. $(\exists x \in \mathbb{N})(\exists y \in \mathbb{N})P(x, y)$

Write down the negation of the statement $(\forall x \in \mathbb{N})(\exists y \in \mathbb{N})P(x, y)$.