Fa'09: MATH 2513–001 | **Discrete Mathematics** | **Noel Brady**

Tuesday 10/27/2009 | Midterm II | 9:00am–10:15am

Name: ____________________ Student ID: ____________________

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	14	
Q2	16	
Q3	20	
Q4	16	
Q5	14	
Q6	20	
TOTAL	100	

Q1]. . . [14 points] Suppose that A, B, C and D are sets. Give definitions of the following: $A \cup B$ and $A \times B$.

Prove that $(A \times B) \cup (C \times D) \subset (A \cup C) \times (B \cup D)$ and give an example to show that the inclusion need not be an equality.

Q2]. . . [16 points] Give the definition of an *injective* function $f : A \to B$.

Give the definition of a *surjective* function $f : A \to B$.

Give the definition of a *bijective* function $f : A \to B$.

Suppose that the composite $f \circ g$ is bijective. Answer the following questions, either giving a reason for your affirmative answer or giving an example to support a negative answer.

1. Must f be injective?

2. Must f be surjective?

3. Must g be injective?

4. Must g be surjective?

Q3]...[20 points] Let $f : A \to B$ be a function, and $S \subset A$ and $T \subset B$ be subsets. Define the *image* $f(S)$ of the subset S and the *preimage* $f^{-1}(T)$ of the subset T.

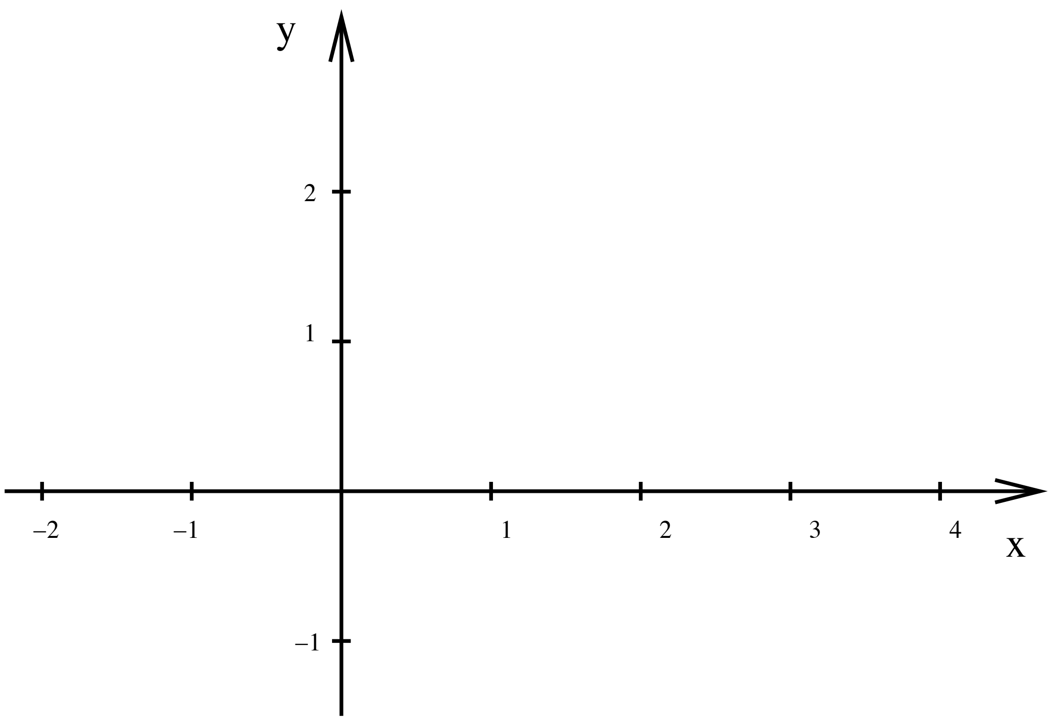
Prove that $f(f^{-1}(T)) \subset T$ for any subset $T \subset B$.

Given an example of a function f and a subset T of the codomain, which shows that the above inclusion need not be an equality.

Give a proof that the inclusion is in fact an equality in the case when f is surjective.

Q4]. . . [16 points] Let $\pi : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto x$ be the projection onto the first coordinate map. Is π injective? Is π surjective?

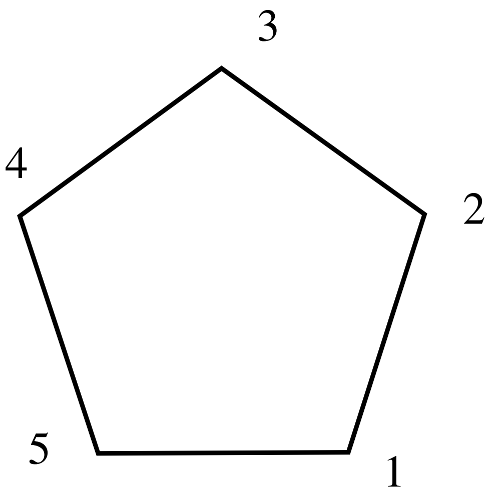
Let $[1, 2]$ denote the interval $\{x \in R | 1 \leq x \leq 2\}$ in $\mathbb{R}$. Draw the set $[1, 2] \times [1, 2] \subset \mathbb{R}^2$.



Draw the preimage $\pi^{-1}(\pi([1, 2] \times [1, 2]))$ in the diagram above. Is it true that

$$\pi^{-1}(\pi([1, 2] \times [1, 2])) \;=\; [1, 2] \times [1, 2]?$$

Q5]. . . [14 points] How many symmetries does the regular pentagon shown have? List these symmetries.



Using the effect that each symmetry has on the vertices of the pentagon, describe a correspondence between the symmetries of the pentagon above and elements of $\mathrm{Perm}(\{1, 2, 3, 4, 5\})$.

Q6]...[20 points] True or false.

1. The power set of a finite set A has $|A|^2$ elements.

2. The set of all functions from a finite set A to itself has $|A|^{|A|}$ elements.

3. If χ_A denotes the characteristic function of a set A, then $\chi_{A\cap B} = \chi_A \chi_B$.

4. $\overline{A \cup B} = \overline{A} \cup \overline{B}$

5. If f and g are bijective and $f \circ g$ is defined, then $(f \circ g)^{-1} = f^{-1} \circ g^{-1}$.

6. A function $f : A \to B$ is an element of the power set of $A \times B$.