If $a \equiv b \pmod{m}$ and $x \equiv y \pmod{m}$,

then $ax \equiv by \pmod{m}$.

<u>Proof</u> By defn of $\equiv \pmod{m}$,

$a \equiv b \pmod{m}$ means $m \mid (b-a)$

i.e. $(b-a) = mq$ for some $q \in \mathbb{Z}$

$\Rightarrow b = a + mq$ for some $q \in \mathbb{Z}$.

Similarly $x \equiv y \pmod{m} \Rightarrow m \mid (y-x)$

$\Rightarrow y - x = mk$ some $k \in \mathbb{Z}$

$\Rightarrow y = x + mk$ some $k \in \mathbb{Z}$

Therefore

$$
\begin{aligned}
by &= (a+mq)(x+mk) \\
&= ax + amk + xmq + m^2kq \\
&= ax + m(\underbrace{ak + xq + mkq}_{\in \mathbb{Z}})
\end{aligned}
$$

$\Rightarrow m \mid (by - ax)$

$\Rightarrow ax \equiv by \pmod{m}$

$\blacksquare$

If $a \equiv b \pmod{m}$ and $x \equiv y \pmod{m}$

then

$$a + x \equiv b + y \pmod{m}.$$

Proof By def$^{\underline{n}}$, $a \equiv b \pmod{m} \Rightarrow m \mid (b-a)$

$\Rightarrow (b-a) = mq$ for some $q \in \mathbb{Z}$

$\Rightarrow b = a + mq$.

Similarly $x \equiv y \pmod{m} \Rightarrow m \mid (y-x)$

$\Rightarrow y - x = mk$ for som $k \in \mathbb{Z}$

$\Rightarrow y = x + mk$

$\Rightarrow b + y = (a + mq) + (x + mk)$

$= (a+x) + m(\underbrace{q+k}_{\text{in } \mathbb{Z}})$

$\Rightarrow m \mid ((b+y) - (a+x))$

$\Rightarrow (a+x) \equiv (b+y) \pmod{m}$

$\boxtimes$
