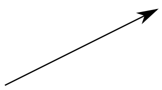
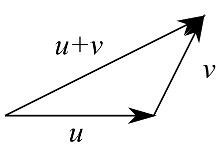
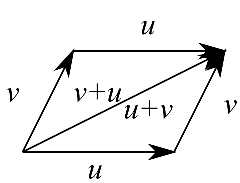
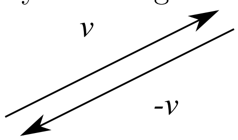
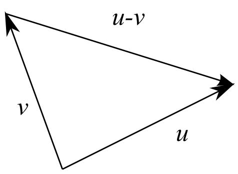
Vectors – Comparing geometry and algebra.

<table>
<tr><th>Geometry</th><th>Algebra in 2-dims</th><th>Algebra in 3-dims</th></tr>
<tr><td>Vector
$\mathbf{v}$ is represented by a
directed line segment.

Any two line segments which are in same direction and of same length represent the same vector.</td><td>$\mathbf{v} = \langle v_1, v_2 \rangle$

The directed line segment connects the initial point $(0, 0)$ to the terminal point (v_1, v_2).</td><td>$\mathbf{v} = \langle v_1, v_2, v_3 \rangle$

The directed line segment connects the initial point $(0, 0, 0)$ to the terminal point (v_1, v_2, v_3).</td></tr>
<tr><td>Addition of vectors
Triangle Law for $\mathbf{u} + \mathbf{v}$

</td><td>Coordinate-wise addition
$\langle u_1, u_2 \rangle + \langle v_1 + v_2 \rangle =$
$\langle u_1 + v_1, u_2 + v_2 \rangle$</td><td>Coordinate-wise addition
$\langle u_1, u_2, u_3 \rangle + \langle v_1, v_2, v_3 \rangle =$
$\langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$</td></tr>
<tr><td>Addition is commutative
$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
is seen using a parallelogram.

</td><td>$\langle u_1, u_2 \rangle + \langle v_1, v_2 \rangle =$
$\langle u_1 + v_1, u_2 + v_2 \rangle =$
$\langle v_1 + u_1, v_2 + u_2 \rangle =$
$\langle v_1, v_2 \rangle + \langle u_1, u_2 \rangle$</td><td>$\langle u_1, u_2, u_3 \rangle + \langle v_1, v_2, v_3 \rangle =$
$\langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle =$
$\langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle =$
$\langle v_1, v_2, v_3 \rangle + \langle u_1, u_2, u_3 \rangle$</td></tr>
<tr><td>Scalar multiplication
$c\mathbf{u}$ is defined by
rescaling for $c > 0$
and also reversing the
direction for $c < 0$</td><td>$c\langle u_1, u_2 \rangle = \langle cu_1, cu_2 \rangle$</td><td>$c\langle u_1, u_2, u_3 \rangle = \langle cu_1, cu_2, cu_3 \rangle$</td></tr>
<tr><td>The zero vector
$\mathbf{0}$ has no length
and arbitrary (no) direction</td><td>$\mathbf{0} = \langle 0, 0 \rangle$</td><td>$\mathbf{0} = \langle 0, 0, 0 \rangle$</td></tr>
</table>

Geometry	**Algebra in 2-dims**	**Algebra in 3-dims**
The negative of a vector $-\mathbf{v}$ is obtained from $\mathbf{v}$ by reversing the direction 	$-\langle v_1, v_2\rangle = \langle -v_1, -v_2\rangle$	$-\langle v_1, v_2, v_3\rangle = \langle -v_1, -v_2, -v_3\rangle$
Difference vector $\mathbf{u}-\mathbf{v}$ connects tip of $\mathbf{v}$ to tip of $\mathbf{u}$. $\mathbf{v}+(\mathbf{u}-\mathbf{v})=\mathbf{u}$ 	$\langle u_1-v_1, u_2-v_2\rangle$ connects (v_1, v_2) to (u_1, u_2).	$\langle u_1-v_1, u_2-v_2, u_3-v_3\rangle$ connects (v_1, v_2, v_3) to (u_1, u_2, u_3).
Components $\mathbf{u}=u_1\mathbf{i}+u_2\mathbf{j}$ $\mathbf{u}=u_1\mathbf{i}+u_2\mathbf{j}+u_3\mathbf{k}$	$\langle u_1, u_2\rangle = u_1\langle 1,0\rangle + u_2\langle 0,1\rangle$	$\langle u_1, u_2\rangle = u_1\langle 1,0,0\rangle + u_2\langle 0,1,0\rangle + u_3\langle 0,0,1\rangle$
Magnitude $\lvert\mathbf{u}\rvert$ is the length of any line segment representing $\mathbf{u}$.	$\lvert\langle u_1, u_2\rangle\rvert = \sqrt{u_1^2+u_2^2}$	$\lvert\langle u_1, u_2, u_3\rangle\rvert = \sqrt{u_1^2+u_2^2+u_3^2}$
Dot Product $\mathbf{u}\cdot\mathbf{v}=\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert\cos\theta$ θ is radian measure of the angle between $\mathbf{u}$ and $\mathbf{v}$.	$\langle u_1, u_2\rangle\cdot\langle v_1, v_2\rangle = u_1v_1+u_2v_2$	$\langle u_1, u_2, u_3\rangle\cdot\langle v_1, v_2, v_3\rangle = u_1v_1+u_2v_2+u_3v_3$
Cross Product (3-dim only) $(\mathbf{u}\times\mathbf{v})\perp\mathbf{u}$ and $(\mathbf{u}\times\mathbf{v})\perp\mathbf{v}$ $\{\mathbf{u},\mathbf{v},\mathbf{u}\times\mathbf{v}\}$ forms a right-handed system, and $\lvert\mathbf{u}\times\mathbf{v}\rvert=\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert\sin\theta$		$\langle u_1, u_2, u_3\rangle\times\langle v_1, v_2, v_3\rangle = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$ $= \langle u_2v_3-u_3v_2, u_3v_1-u_1v_3, u_1v_2-u_2v_1\rangle$

Properties of vector addition and scalar multiplication

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
3. $\mathbf{u} + \mathbf{0} = \mathbf{u}$
4. $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
5. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$
6. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$
7. $c(d\mathbf{u}) = (cd)\mathbf{u}$
8. $1\mathbf{u} = \mathbf{u}$