HWK 12

pg 524

(10) $\int_0^{\pi} \cos^6\theta \, d\theta = \int_0^{\pi} (\cos^2\theta)^3 d\theta$

$= \frac{1}{8}\int_0^{\pi} (1+\cos 2\theta)^3 d\theta$

$= \frac{1}{8}\int_0^{\pi} (1 + 3\cos 2\theta + 3\cos^2 2\theta + \cos^3 2\theta) d\theta$

$= \frac{1}{8}\left[\theta\Big|_0^{\pi} + \frac{3}{2}\sin 2\theta\Big|_0^{\pi} + \frac{3}{2}\int_0^{\pi}(1+\cos 4\theta)d\theta + \int_0^{\pi}(1-\sin^2 2\theta)\cos 2\theta \, d\theta\right)$

$u = \sin 2\theta$ $\quad \cancel{x}=0 \to u=0$

$du = 2\cos 2\theta$ $\quad x=\pi \to u=0$

$= \frac{1}{8}\left[(\pi-0)+(\frac{3}{2})(\underset{0}{\sin 2\pi} - \underset{0}{\sin 0}) + \frac{3}{2}\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\pi} + \frac{1}{2}\int_0^{0}(1-u^2)du\right)$

$= \frac{1}{8}\left[\pi \;\; \frac{3}{2}\left[\pi + \frac{1}{4}\underset{0}{\sin 4\pi} - (0 + \frac{1}{4}\underset{0}{\sin 0})\right] + 0\right]$

$= \frac{1}{8}\left[\pi + \frac{3}{2}\pi\right] = \boxed{\frac{5\pi}{16}}$

(22) $\int_0^{\pi/2} \sec^4(t/2)dt = 2\int_0^{\pi/4} \sec^4(u)du$

$u = t/2$ $\quad t=0 \to u=0$

$du = \frac{1}{2}t dt$ $\quad t=\pi/2 \to u = \pi/4$

$= 2\int_0^{\pi/4}(\tan^2 u + 1)\sec^2 u \, du$

$= 2\int_0^{1}(w^2+1)dw$

$w = \tan u$ $\quad u=0 \to w=0$

$dw = \sec^2 u \, du$ $\quad u=\pi/4 \to w=1$

$= 2\left[\frac{w^3}{3} + w\right]_0^1$

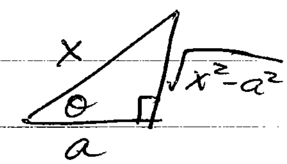
$= 2\left[\frac{1}{3} + 1 - (0+0)\right]$

$= \boxed{\frac{8}{3}}$

HWK 12 cont.

Pg 530.

(8) $\int \frac{\sqrt{x^2-a^2}}{x^4}\,dx$

$x = a\sec\theta \longrightarrow \sec\theta = \frac{x}{a} \longrightarrow$ 

$dx = a\sec\theta\tan\theta\,d\theta$

$$\sqrt{x^2-a^2} = \sqrt{a^2\sec^2\theta - a^2} = a\sqrt{1-\sec^2\theta}$$
$$= a\sqrt{\tan^2\theta} = a\tan\theta$$

$$\int \frac{\sqrt{x^2-a^2}}{x^4}\,dx = \int \frac{a\tan\theta}{a^4\sec^4\theta}\cdot a\sec\theta\tan\theta\,d\theta$$
$$= \frac{1}{a^2}\int \frac{\tan^2\theta}{\sec^3\theta}\,d\theta$$
$$= \frac{1}{a^2}\int \frac{\sin^2\theta}{\cancel{\cos^2\theta}}\cdot\frac{\cos^3\theta}{1}\,d\theta$$
$$= \frac{1}{a^2}\int \sin^2\theta\cos\theta\,d\theta$$

$u = \sin\theta$
$du = \cos\theta\,d\theta$

$$= \frac{1}{a^2}\int u^2\,du$$
$$= \frac{1}{a^2}\,u^3 + C$$
$$= \frac{1}{a^2}\cdot\frac{1}{3}\sin^3\theta + C$$
$$= \frac{1}{3a^2}\left(\frac{\sqrt{x^2-a^2}}{x}\right)^3 + C$$

(26) $\int \frac{x^2}{\sqrt{4x-x^2}}\,dx$

1st we have to complete the square.

HWK 12 ctd.

26 ctd.

$$4x - x^2 = -(\underline{x^2 - 4x + 4} - 4)$$
$$\left(\tfrac{-4}{2}\right)^2$$

$$= -(x^2 - 4x + 4) + 4$$

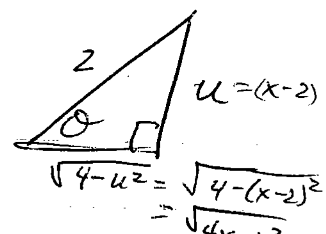
$$= -(x-2)^2 + 4 = 4 - (x-2)^2$$

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,dx = \int \frac{x^2}{\sqrt{4-(x-2)^2}} = \int \frac{(u+2)^2}{\sqrt{4-u^2}}\,du$$

u-sub. $u = x-2 \rightarrow x = u+2$.
$du = dx$

Now we do a trig sub.

$u = 2\sin\theta \rightarrow \sin\theta = \frac{u}{2} \longrightarrow$



$du = 2\cos\theta\, d\theta$

$\sqrt{4-u^2} = \sqrt{4-4\sin^2\theta} = 2\cos\theta$

$$\int \frac{x^2}{\sqrt{4x^2-x^2}}\,dx = \int \frac{(2\sin\theta+2)^2}{\cancel{2\cos\theta}} \cdot \cancel{2\cos\theta}\, d\theta$$

$$= \int (2^2\sin^2\theta + 8\sin\theta + 4)\,d\theta$$

$$= 4 \cdot \tfrac{1}{2}\int(1-\cos 2\theta)\,d\theta + 8\int \sin\theta\,d\theta + \int 4\,d\theta$$

$$= 2\left[\theta - \tfrac{1}{2}\sin 2\theta\right] + 8(-\cos\theta) + 4\theta + C$$

use identity $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 6\theta - \sin 2\theta - 8\cos\theta + C$$

$$= 6\theta - 2\sin\theta\cos\theta - 8\cos\theta + C$$

From

$\sin\theta = \frac{x-2}{2}$

$\cos\theta = \frac{\sqrt{4x-x^2}}{2}$

$\theta = \sin^{-1}\left(\frac{x-2}{2}\right)$

$$= 6\sin^{-1}\left(\frac{x-2}{2}\right) - 2\left(\frac{x-2}{2}\right)\left(\frac{\sqrt{4x-x^2}}{2}\right) - 8\left(\frac{\sqrt{4x-x^2}}{2}\right) + C$$

$$\boxed{= 6\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{1}{2}(x-2)\sqrt{4x-x^2} - 4\sqrt{4x-x^2} + C}$$

Hwk 12 ctd.

Pg 510

(8) $\int \frac{r^2}{r+4}\,dr$

Long division needed

$$\begin{array}{r} r-4 \\ r+4\ \overline{)\ r^2+0r+0} \\ -(r^2+4r) \\ \hline -4r \\ -(-4r-16) \\ \hline 16 \end{array}$$

$$\int \frac{r^2}{r+4}\,dr = \int r-4+\frac{16}{r+4}\,dr$$

$u = r+4$
$du = dr$

$$= \frac{r^2}{2} - 4r + 16\ln|r+4| + C$$

(20) $\int \frac{x^2}{(x-3)(x+2)^2}\,dx$

First decompose & solve

$$\frac{x^2}{(x-3)(x+2)^2} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$x^2 = A(x+2)^2 + B(x-3)(x+2) + C(x-3)$$

for $x=3 \to 3^2 = A(3+2)^2 + 0 + 0 = 25A \Rightarrow A = \frac{9}{25}$

for $x=-2 \to (-2)^2 = 0 + 0 + C(-2-3) = -5C \Rightarrow C = \frac{-4}{5}$

$$x^2 = \underline{Ax^2} + 2Ax + 4A + \underline{Bx^2} - Bx - 6B + Cx - 18$$

$A + B = 1 \Rightarrow B = \frac{16}{25}$

$$\int \frac{x^2}{(x-3)(x+2)^2}\,dx = \frac{9}{25}\int \frac{1}{x-3}\,dx + \frac{16}{25}\int \frac{1}{x+2}\,dx + \frac{-4}{5}\int \frac{1}{(x+2)^2}\,dx$$

$$= \frac{9}{25}\ln|x-3| + \frac{16}{25}\ln|x+2| - \frac{4}{5}\cdot\frac{1}{(x+2)} + C$$