Homework set #8 (Solution)

#20, $h(x) = \ln(x + \sqrt{x^2-1})$

$$h'(x) = \frac{d[\ln(x+\sqrt{x^2-1})]}{d(x+\sqrt{x^2-1})} \cdot \frac{d(x+\sqrt{x^2-1})}{dx} \quad \text{(Chain rule)}$$

$$= \left(\frac{1}{(x+\sqrt{x^2-1})}\right) \cdot \left(1 + \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-1}}\right)$$

$$= \frac{1}{\cancel{x+\sqrt{x^2-1}}} \cdot \frac{(\cancel{\sqrt{x^2-1}+x})}{\sqrt{x^2-1}} = \boxed{\frac{1}{\sqrt{x^2-1}}}$$

26. $G(u) = \ln\sqrt{\frac{3u+2}{3u-2}} = \ln\left(\frac{3u+2}{3u-2}\right)^{\frac{1}{2}} = \frac{1}{2}\ln\left(\frac{3u+2}{3u-2}\right)$

$$= \frac{1}{2}\left[\ln 3u+2 - \ln(3u-2)\right]$$

$$\therefore G'(u) = \frac{1}{2}\left[\frac{1}{3u+2}\cdot 3 - \frac{1}{3u-2}\cdot 3\right]$$

$$= \frac{3}{2}\left[\frac{\cancel{3u}-2-\cancel{3u}-2}{(9u^2-4)}\right]$$

$$= \boxed{\frac{-6}{9u^2-4}}$$

#30. $y = \ln|\tan 2x|$

$$y' = \frac{2\sec^2 2x}{\tan 2x}.$$

#56. $y = \dfrac{(x^3+1)^4 \sin^2 x}{\sqrt[3]{x}}$

Taking ln on both sides,

we get, $\ln y = \ln\left[\dfrac{(x^3+1)^4 \sin^2 x}{\sqrt[3]{x}}\right]$

$$= 4\ln|x^3+1| + 2\ln|\sin x| - \frac{1}{3}\ln|x|.$$

$$\frac{y'}{y} = 4\frac{3x^2}{x^3+1} + \frac{2\cos x}{\sin x} - \frac{1}{3x}$$

$$\Rightarrow y' = \left(\frac{12x^2}{x^3+1} + 2\cot x - \frac{1}{3x}\right)y.$$

$$= \boxed{\frac{(x^3+1)^4 \sin^2 x}{\sqrt[3]{x}}\left[\frac{12x^2}{x^3+1} + 2\cot x - \frac{1}{3x}\right]}$$

#58. $y = \sqrt[4]{\dfrac{x^2+1}{x^2-1}}$

$$\ln y = \ln\left(\sqrt[4]{\frac{x^2+1}{x^2-1}}\right)$$

$$= \frac{1}{4}\ln(x^2+1) - \frac{1}{4}\ln(x^2-1).$$

$$\frac{1}{y}y' = \frac{1 \cdot 2x}{4(x^2+1)} - \frac{2x}{4(x^2-1)}$$

$$\Rightarrow y' = \left[\frac{2x\,(\cancel{x^2}-1-\cancel{x^2}-1)}{4(x^4-1)}\right] y$$

$$= \frac{-\cancel{4}x}{\cancel{4}(x^4-1)}\, y = \boxed{-\frac{x}{(x^4-1)}\left(\sqrt[4]{\frac{x^2+1}{x^2-1}}\right)}$$

60. $\displaystyle\int_1^2 \frac{4+u^2}{u^3}\,du$

$$= \int_1^2 (4u^{-3} + u^{-1})\,du.$$

$$= \left(\frac{4}{-2}u^{-2} + \ln|u|\right)\Big|_1^2 = \left(-\frac{2}{u^2} + \ln|u|\right)\Big|_1^2$$

$$= -\frac{1}{2} + \ln 2 - (-2 + \ln 1)$$

$$= \boxed{\frac{3}{2} + \ln 2}$$

64. $\displaystyle\int_e^6 \frac{dx}{x\ln x}$

Let $u = \ln x$ $\quad du = \frac{1}{x}dx$

when $x = e$, $u = \ln e = 1$

" $x = 6$ $\quad u = \ln 6.$

$$\int_1^{\ln 6} \frac{1}{u}\,du = \left[\ln|u|\right]_1^{\ln 6} = \ln(\ln 6) - \ln 1$$

$$= \boxed{\ln(\ln 6)}$$

#66, $\int \frac{\cos x}{2+\sin x}\,dx.$

Let $u = 2+\sin x$

$du = \cos x\,dx.$

$= \int \frac{1}{u}\,du = \ln|u|+C$

$= \ln|2+\sin x| + C.$

$= \boxed{\ln(2+\sin x)+C}$ since $\boxed{2+\sin x > 0}$

#67 $\int \frac{(\ln x)^2}{x}\,dx$

Let $u = \ln x$

$du = \frac{1}{x}\,dx.$

$= \int u^2\,du$

$= u^3/3 + C = \boxed{\frac{1}{3}(\ln x)^3 + C}$