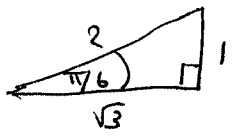
Pge 187



Q2 $f(x) = \sin(x) \qquad a = \pi/6$

$f'(x) = \cos(x)$

$\Rightarrow \begin{cases} f(a) = \sin(\pi/6) = \frac{1}{2} \\ f'(a) = \cos(\pi/6) = \sqrt{3}/2 \end{cases}$

$$L(x) = f(a) + f'(a)(x-a) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$$

$$\boxed{L(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)}$$

Q11 (a) $y = x^2 \sin(2x)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 \sin(2x)\right) = \frac{dx^2}{dx}\sin(2x) + x^2 \frac{d}{dx}\{\sin(2x)\}$$

$$= 2x \sin(2x) + x^2 \cos(2x) \cdot \frac{d(2x)}{dx} \quad \leftarrow 2$$

$$= 2x \sin(2x) + 2x^2 \cos(2x)$$

$$\boxed{dy = \left(\frac{dy}{dx}\right)dx = \left(2x \sin(2x) + 2x^2 \cos(2x)\right)dx}$$

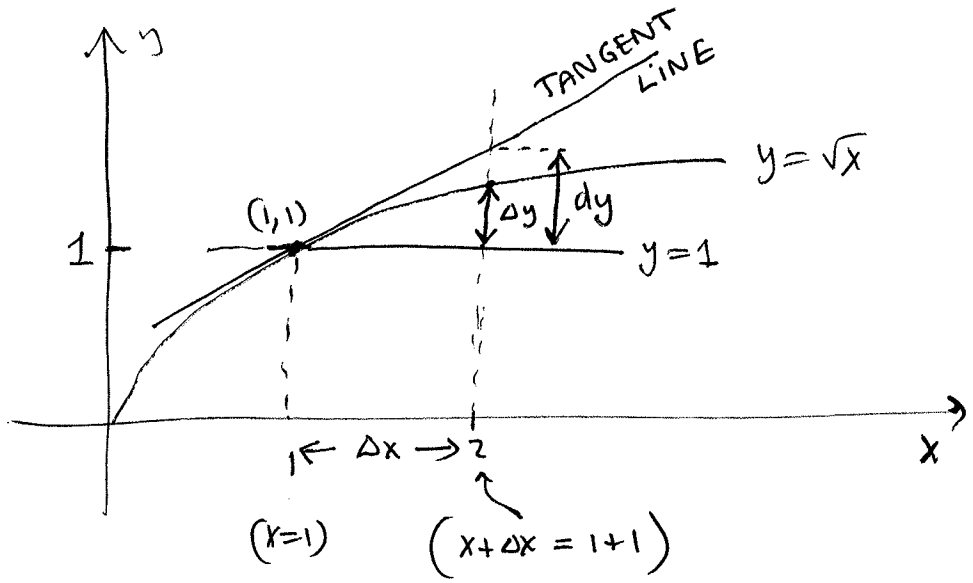
(b) $y = \sqrt{1+t^2}$

$$\frac{dy}{dt} = \frac{d}{dt}\left(\sqrt{1+t^2}\right) = \frac{1}{2}(1+t^2)^{\frac{1}{2}-1} \cdot \frac{d}{dt}(1+t^2)$$

$$= \frac{1}{2\sqrt{1+t^2}} \cdot 2t = \frac{t}{\sqrt{1+t^2}}$$

$$\boxed{dy = \left(\frac{dy}{dt}\right)dt = \frac{t\,dt}{\sqrt{1+t^2}}}$$

Q20



$$\Delta y = y(x+\Delta x) - y(x)$$

$$= \sqrt{1+1} - \sqrt{1}$$

$$= \sqrt{2} - 1$$

$$\simeq 1.4142\ldots - 1$$

$$= 0.4142\ldots$$

$\left(x=1,\ \Delta x=1 = dx\right)$

$$dy = \frac{dy}{dx}dx = \frac{d\sqrt{x}}{dx}dx = \frac{1}{2\sqrt{x}}dx$$

$$= \frac{1}{2\sqrt{1}} \cdot (1)$$

$$= \frac{1}{2}$$

$$= 0.5$$

Q25 [Using Lin. Approx.]

$f(x) = \sqrt[3]{x}$ $\qquad f(1000) = \sqrt[3]{1000} = 10$

input is 1000 ; $\Delta x = 1$. $\qquad f'(x) = \frac{1}{3}x^{-2/3}$

$f'(1000) = \frac{1}{3(10)^2}$

$$L(x) = f(1000) + f'(1000)(x - 1000)$$

$$= 10 + \frac{1}{3(10)^2}(x - 1000)$$

$$\sqrt[3]{1001} = f(1001) \approx L(1001)$$

$$= 10 + \frac{1}{3(10)^2}(1001 - 1000)$$

$$= 10 + \frac{1}{300} = 10 + 0.00333\ldots$$

$$= 10.00333\ldots$$

Q25 [USING DIFFERENTIALS] $\qquad dy = \frac{dy}{dx}dx = \frac{d\sqrt[3]{x}}{dx}dx$

$$= \frac{1}{3x^{2/3}}dx$$

$x = 1000$; $dx = 1$

$$\Rightarrow dy = \frac{1}{3(1000)^{3/2}} \cdot 1 = \frac{1}{3(10)^2} = \frac{1}{300}$$

$$\sqrt[3]{1001} = f(1001) = f(1000) + (f(1001) - f(1000))$$

$$= \sqrt[3]{1000} + \Delta y = 10 + \Delta y$$

$$\approx 10 + dy = 10 + \frac{1}{300} = 10.00333\ldots$$

Q30 $f(x) = x^6$ $x = 1$ $dx = \Delta x = 0.01$

$$dy = \frac{dy}{dx}\,dx = 6x^5\,dx$$ when $x=1$ & $dx = 0.01$ we get

$$dy = 6(1)^5\,0.01 = 0.06$$

$$(1.01)^6 = f(1.01) = f(1) + \big(f(1.01) - f(1)\big)$$
$$= f(1)^6 + \Delta y$$
$$= 1 + \Delta y$$
$$\approx 1 + dy = 1 + 0.06 = \boxed{1.06}$$

Q34 Volume of $\frac{1}{2}$ ball of radius r is $V(r) = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$

$\boxed{V(r) = \frac{2}{3}\pi r^3}$ $\boxed{\frac{dV}{dr} = 2\pi r^2}$

r starts at $\frac{50}{2}$ m $= 25$ m ← watch units!

& increases by an amount $dr = 0.05$ cm $= \frac{0.05}{100}$ m

$= 0.0005$ m

Volume of paint needed

$$= V(25 + dr) - V(25)$$
$$= \Delta V$$
$$\approx dV = \frac{dV}{dr}\,dr = 2\pi(25)^2(0.0005)\ m^3$$
$$= \frac{10\pi}{16}\ M^3 \text{ of paint.}$$