Monday 12/09/2013 Final Examination 120 minutes

Name: Student ID:

Instructions.

1. Attempt all questions.
2. Do not write on back of exam sheets. Extra paper is available if you need it.
3. Show all the steps of your work clearly.

Question	Points	Your Score
Q1	10	
Q2	15	
Q3	15	
Q4	15	
Q5	15	
Q6	15	
Q7	15	
Q8	15	
Q9	10	
Q10	10	
Q11	10	
TOTAL	100	

Q1]. . . [10 points] Compute the following derivatives.
Find $f'(x)$ where

$$f(x) \;=\; \sqrt{x}\tan x$$

Find the derivative y' where

$$y \;=\; \sin^4\Big(\frac{x}{2x-1}\Big)$$

Find the 2013^{th} derivative — $g^{(2013)}(x)$ — where

$$g(x) \;=\; x^{2013} + \cos(2x-9)$$

Q2]. . . [15 points] Find the following finite or infinite limits.

- $\lim_{x \to 2} \dfrac{(x+3)^2 - 25}{x - 2}$

- $\lim_{x \to \infty} \dfrac{4 - 3x - x^3}{(2x-1)^3}$

- $\lim_{\theta \to 0} \dfrac{\sin 3\theta}{\theta + \tan \theta}$

Q3]. . . [15 points] Evaluate the following integrals.

- $$\int \frac{x}{\sqrt{x^2-1}}\,dx$$

- $$\int_3^0 \sqrt{9-x^2}+x^2-2\,dx$$

Q4]. . . [15 points] Find the rectangle of maximum area which has its base on the x-axis and its two upper corners on the graph of $y = 12 - x^2$.

- Draw the region enclosed by the x-axis and the graph of $y = 12 - x^2$, and draw in a typical inscribed rectangle.

- Find the dimensions (height, width, and area) of the rectangle above which has maximum area. Show all the steps of your work.

Q5]. . . [15 points] A fuel tank in the shape of an inverted right circular cone is being filled at the rate of 2 cubic feet per minute. The height of the cone is 16 feet and the radius 4 feet. How fast is the fuel level rising when the fuel is 5 feet deep?

Q6]. . . [15 points] Find the equation of the tangent line to the curve $x^2y - y^3 = 8$ at the point $(3, -1)$.

Q7]. . . [15 points] Compute the area of the region bounded by the curves

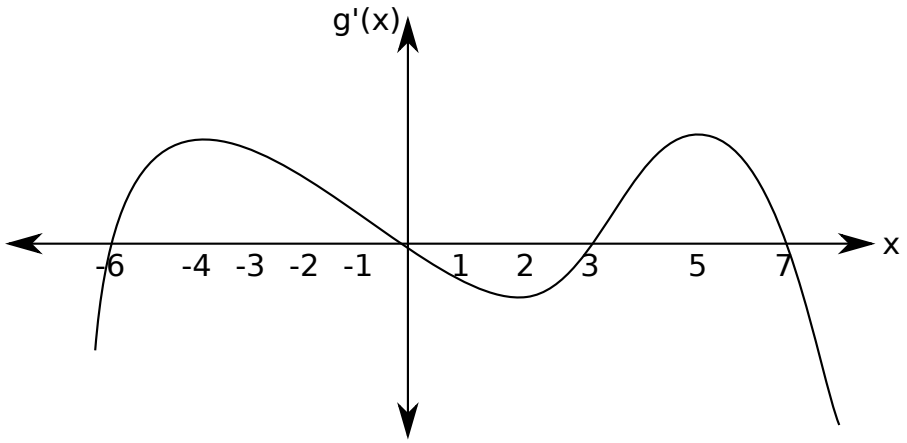
$$y = \frac{1}{\sqrt{x}}, \quad y = \frac{1}{x^2}, \quad \text{and } x = 4.$$

You may find it helpful to sketch the region.

Q8]. . . [15 points] Write down the linearization $L(x)$ of the function $f(x) = \sqrt[3]{x}$ at the point $a = 4$.

Use the linearization above to estimate the value of $\sqrt[3]{64.048}$.

Q9]. . . [10 points] The following is a graph of $g'(x)$. Answer the following questions about the antiderivative $g(x)$.



Find all of the intervals on which the function $g(x)$ is increasing.

Find all of the intervals on which the function $g(x)$ is decreasing.

State the x-values of any local extreme points for $g(x)$, specifying whether they are local maxima or local minima.

Q10]. . . [10 points] Compute the following limit, showing your work carefully:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \Big(\frac{7i^2}{n^2} + \frac{5}{n} \Big).$$

The following expression is a Riemann sum which estimates the area under the curve of a function $f(x)$ on an interval $[a, b]$ using rectangles constructed from the left-hand endpoints. Find $f(x)$ and $[a, b]$. You do NOT need to compute the exact area.

$$L_n = \sum_{i=1}^{n} 3\sqrt{7 + \frac{4(i-1)}{n}} \cdot \frac{4}{n}$$