

VECTOR FIELDS & FUNCTIONS (2-dims)



$$f(x, y) \rightsquigarrow \nabla f = \langle f_x, f_y \rangle$$

$$\vec{F} = \langle P, Q \rangle \rightsquigarrow (Q_x - P_y)$$

$$f(\vec{r}(b)) - f(\vec{r}(a)) = \int_C \nabla f \cdot d\vec{r}$$

FUND. THM. OF PATH INTEGRALS

$$\oint_{\partial D} \vec{F} \cdot d\vec{r} = \iint_D \hat{k} \cdot \text{curl}(\vec{F}) dA$$

$$\oint_{\partial D} P dx + Q dy = \iint_D (Q_x - P_y) dA$$

GREEN'S THM

- $\hat{k} \cdot \text{curl}(\text{grad}(f)) = 0$ always
- If $\hat{k} \cdot \text{curl}(\vec{F}) = 0$, then \vec{F} can be locally integrated to give a scalar f with $\nabla f = \vec{F}$.

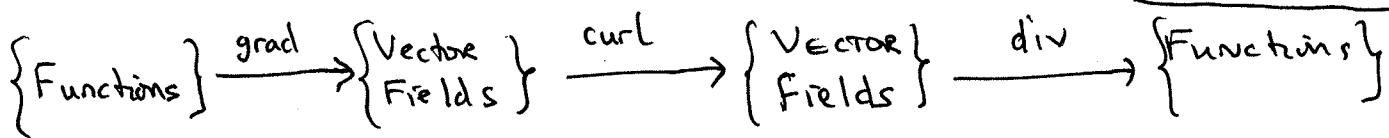
However $\vec{F} = \frac{\langle -y, x \rangle}{x^2 + y^2}$ satisfies $\hat{k} \cdot \text{curl}(\vec{F}) = 0$

yet $\vec{F} \neq \nabla f$ for a globally defined function on $\mathbb{R}^2 - \{(0,0)\}$.

↓
because $\oint_C \vec{F} \cdot d\vec{r} = 2\pi$ where $C: x^2 + y^2 = 1$

VECTOR FIELDS & FUNCTIONS (3-dim)

$$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$



$f \rightsquigarrow \nabla f = \langle f_x, f_y, f_z \rangle$

$\vec{G} = \langle P, Q, R \rangle \rightsquigarrow \nabla \times \vec{G} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$

$\vec{F} = \langle P, Q, R \rangle \rightsquigarrow \nabla \cdot \vec{F} = P_x + Q_y + R_z$

$$f(\vec{r}(b)) - f(\vec{r}(a)) = \int_C \nabla f \cdot d\vec{r}$$

FUND. THM. OF PATH INTEGRALS

$$\oint_{\partial S} \vec{G} \cdot d\vec{r} = \iint_S \text{curl}(\vec{G}) \cdot d\vec{S}$$

STOKES' THM.

$$\oint_{\partial E} \vec{F} \cdot d\vec{S} = \iiint_E \text{div}(\vec{F}) dV$$

DIVERGENCE THM.

• $\nabla \times (\nabla f) = \vec{0}$ always

• $\nabla \cdot (\nabla \times \vec{G}) = 0$ always

• If $\nabla \times \vec{F} = \vec{0}$, then \vec{F} can be locally integrated to give f with $\nabla f = \vec{F}$.

• If $\nabla \cdot \vec{F} = 0$, then \vec{F} can be locally integrated to give \vec{G} with $\nabla \times \vec{G} = \vec{F}$.

• NOT GLOBAL: $\vec{F} = \frac{\langle -y, x, 0 \rangle}{x^2 + y^2}$ satisfies $\nabla \times \vec{F} = \vec{0}$, yet $\oint_C \vec{F} \cdot d\vec{r} = 2\pi$

$C: \langle \cos t, \sin t, 0 \rangle$
 $0 \leq t \leq 2\pi$

• NOT GLOBAL: $\vec{F} = \frac{\langle x, y, z \rangle}{(x^2 + y^2 + z^2)^{3/2}}$ satisfies $\nabla \cdot \vec{F} = 0$, yet $\iint_S \vec{F} \cdot d\vec{S} = 4\pi$ $S: x^2 + y^2 + z^2 = 1$