CALCULUS III FALL 1999
HOMEWORK 11 – ANSWERS

§11.1 Question 6,14,16,20; §11.2 Questions 22,40; §11.3 Questions 6,8,24,34,56,58

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6. We have $AB = 3,\ BC = 3,\ AC = \sqrt{26}$. So the triangle is isosceles but not a right triangle.

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14. The equation is

$$(x-1)^2 + (y-2)^2 + (z+3)^2 = 49$$

which is

$$x^2 - 2x + y^2 - 4y + z^2 + 6z = 35.$$

16.

$$x^2 + y^2 + z^2 = 6x + 4y + 10z$$

gives

$$(x-3)^2 + (y-2)^2 + (z-5)^2 = 9 + 4 + 25 = 38,$$

so the centre of the circle is at $(3, 2, 5)$ and its radius is $\sqrt{38}$.

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20. The equation is equivalent to

$$\left(x + \frac{a}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2 + \left(z + \frac{c}{2}\right)^2 + d = \frac{a^2 + b^2 + c^2}{4},$$

which is the equation of circle with centre $(-a/2, -b/2, -c/2)$ and radius

$$\frac{1}{2}\sqrt{a^2 + b^2 + c^2 - 4d}.$$

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22. The vector $\mathbf{v} = \mathbf{i} - 4\mathbf{j} + 8\mathbf{k}$ has length 9 and so the required vector is $\mathbf{v}/9$.

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40. If the vectors $\mathbf{a}$ and $\mathbf{b}$ lie along two sides of the triangle, then $\mathbf{a} - \mathbf{b}$ lies along the third side. The midpoints of the first two sides are at the heads of the vectors $\mathbf{a}/2$ and $\mathbf{b}/2$. So the required result is a consequence of the observation $\mathbf{a}/2 - \mathbf{b}/2 = (\mathbf{a} - \mathbf{b})/2$.

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6. $\mathbf{a} \cdot \mathbf{b} = 1$.

8.

$$\mathbf{a} \cdot \mathbf{b} = 6\left(\frac{1}{3}\right)\cos\frac{\pi}{4} = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

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24. Note that $\mathbf{b} = (-3/2)\mathbf{a}$, so the vectors are parallel.

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34. Note that $|\mathbf{a}| = 5\sqrt{2}$ and so the direction cosines are $3/(5\sqrt{2},\ 1\sqrt{2},\ 4/(5\sqrt{2})$. The direction angles are just the inverse cosines of these – you can use your calculator to approximate these, but note that one of them is $\pi/4$.

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56. The required angle is the same as the angle between $\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j}$. We have $|\mathbf{i} + \mathbf{j} + \mathbf{k}| = \sqrt{3}$ and $|\mathbf{i} + \mathbf{j}| = \sqrt{2}$ and $(\mathbf{i} + \mathbf{j} + \mathbf{k}) \cdot (\mathbf{i} + \mathbf{j}) = 2$. So the required angle is $\cos^{-1}\sqrt{6}/3$.

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58. In order to show that the two angles are equal, it suffices to prove that

$$\mathbf{c} \cdot \frac{\mathbf{a}}{|\mathbf{a}|} = \mathbf{c} \cdot \frac{\mathbf{b}}{|\mathbf{b}|}.$$

We note that

$$\mathbf{c} \cdot \frac{\mathbf{a}}{|\mathbf{a}|} = \mathbf{b} \cdot \mathbf{a} + |\mathbf{a}||\mathbf{b}| = \mathbf{c} \cdot \frac{\mathbf{b}}{|\mathbf{b}|},$$

and so we are finished.