CALCULUS III — FALL 1999

HOMEWORK 12 – ANSWERS

§11.4 Questions 6,24,28,30; §11.5 Questions 8,16,34,38,54,64

6.

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & -1 \\ 3 & -1 & 7 \end{vmatrix} = 13\mathbf{i} - 10\mathbf{j} - 7\mathbf{k}.$$

24.

(a) The plane is parallel to the vectors $4\mathbf{i}+9\mathbf{j}+3\mathbf{k}$ and $7\mathbf{i}+5\mathbf{j}+6\mathbf{k}$. So the required orthogonal vector is

$$(4\mathbf{i}+9\mathbf{j}+3\mathbf{k})\times(7\mathbf{i}+5\mathbf{j}+6\mathbf{k}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 9 & 3 \\ 7 & 5 & 6 \end{vmatrix} = 39\mathbf{i} - 3\mathbf{j} - 43\mathbf{k}.$$

(b) The area of the triangle is $\frac{1}{2}|39\mathbf{i} - 3\mathbf{j} - 43\mathbf{k}| = \frac{1}{2}\sqrt{3379}$.

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28. The parallelepiped is generated by the vectors $2\mathbf{i}+3\mathbf{j}+3\mathbf{k}$, $-\mathbf{i}-\mathbf{j}-\mathbf{k}$ and $6\mathbf{i}-2\mathbf{j}+2\mathbf{k}$. So the volume is

$$\left\|\begin{matrix} 2 & 3 & 3 \\ -1 & -1 & -1 \\ 6 & -2 & 2 \end{matrix}\right\| = 4.$$

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30. These four points can be used to form a parallelepiped, as in the previous question. The points are coplanar (lie in a single plane) precisely when this parallelepiped has zero volume.

The parallelepiped is generated by the vectors $\mathbf{i}+4\mathbf{j}+5\mathbf{k}$, $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $5\mathbf{i}+2\mathbf{j}+7\mathbf{k}$. The volume of the parallelepiped is therefore

$$\left\|\begin{matrix} 1 & 4 & 5 \\ 2 & -1 & 1 \\ 5 & 2 & 7 \end{matrix}\right\| = 0, \text{ as required.}$$

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8. The vector $\mathbf{v} = 4\mathbf{i} - 3\mathbf{j} - (1/2)\mathbf{k}$ is parallel to the line. So the vector equation of the line can be written as

$$\mathbf{r} = -\mathbf{i} + 4\mathbf{j} + \mathbf{k} + t(4\mathbf{i} - 3\mathbf{j} - (1/2)\mathbf{k}).$$

It follows that the parametric eautions are

$$x = -1 + 4t, \qquad y = 4 - 3t, \qquad z = 1 - \frac{t}{2},$$

and the symmetric equations are

$$\frac{x = 1}{4} = -\frac{y-4}{3} = -(2z-2).$$

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16. The parametric equations of the lines are

$$x = 1 + 2t,\ y = t,\ z = 1 + 4t \qquad \text{and} \qquad x = s,\ y = -2 + 2s,\ z = -2 + 3s.$$

Note that putting $t = 0$ and $s = 1$ yields the point of intersection $(1, 0, 1)$.

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34. Note that $\mathbf{v} = 5\mathbf{i} + \mathbf{j} - \mathbf{k}$ is parallel to the line. Also $(0, 1, 0)$ is a point on the line. Hence the plane is also parallel to the vector $\mathbf{w} = -\mathbf{i} - \mathbf{j} + \mathbf{k}$. Consequently $\mathbf{v} \times \mathbf{w} = -4\mathbf{j} - 4\mathbf{k}$ is orthogonal to the plane. It follows that the equation of the plane is $y + z = 1$.

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38. The appropriate parameter value is given by the equation

$$1 + t = 1 - 2 + 2t + t = -1 + 3t \qquad \text{and so} \qquad t = 1.$$

Consequently the point of intersection is $(0, 1, 2)$.

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54. The normals to the given planes are

$$\mathbf{a} = \mathbf{i} - \mathbf{k} \qquad \mathbf{b} = \mathbf{j} + 2\mathbf{k} \qquad \mathbf{c} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}.$$

The required plane contains $\mathbf{a} \times \mathbf{b} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{c}$. So the normal to the plane is $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times \mathbf{c} = 3\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$. Finally note that $(1, 3, 0)$ is a point of the plane and so the equation of the plane is $x + y + z = 4$.

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64. The required distance is

$$\frac{|12 + 12 + 7 - 5|}{\sqrt{16 + 36 + 1}} = \frac{26}{\sqrt{53}}.$$