CALCULUS III FALL 1999

HOMEWORK 4 – ANSWERS

§9.6 Questions 10,14,44; §10.1 Questions 16,18,22,32,38

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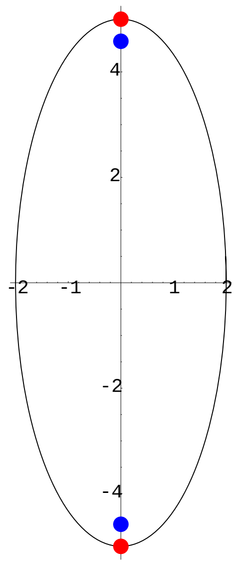
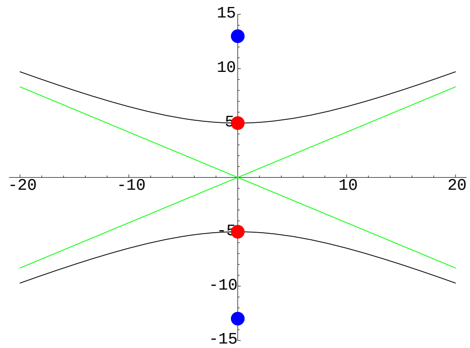
10. The y-axis is the major axis of this ellipse and so its focis are on this axis. The eccentricity satisfies $4 = 25(1 - e^2)$ and so $e = \sqrt{21}/5$. The foci are therefore at $(0, \pm\sqrt{21})$ and the vertices are at $(0, \pm 5)$. The ellipse looks like



14. The major axis of this hyperbola is the y-axis and so its foci are on this axis. The eccentricity satisfies $144 = 25(1 - e^2)$ and so $e = 13/5$. The foci are therefore at $(0, \pm 13)$ and the vertices at $(0, \pm 5)$. The asymptotes are $y = \pm 5x/12$. The hyperbola looks like



44. First we assume that the foci are at $(0, 0)$ and $(-2\sqrt{2}, 0)$ and that the major axis is horizontal with length 4. This is just a rotation and translation of the required ellipse. We have $a = 2$ and $ae = \sqrt{2}$ which gives $e = 1/\sqrt{2}$ and $\ell = a(1 - 1/2) = 1$. So the polar equation of this ellipse is

$$r = \frac{1}{1 + (\cos\theta)/\sqrt{2}}.$$

Applying a rotation of $\pi/4$ about the origin will result in an ellipse with foci at the origin and $(-2, -2)$. This ellipse will have polar equation

$$r = \frac{1}{1 + (\cos(\theta - \pi/4))/\sqrt{2}} = \frac{2}{2 + \cos\theta + \sin\theta}.$$

The cartesian equivalent is obtained by multiplying the right side numerator and denominator by r. This gives $2\sqrt{x^2 + y^2} + x + y = 2$. Finally, we translate 1 unit to the right and 1 unit vertically to obtain the desired ellipse. This must have equation

$$2\sqrt{(x-1)^2 + (y-1)^2} + (x - 1) + (y - 1) = 2$$

which is equivalent to $3x^2 - 2xy + 3y^2 = 8$.

16.

$$a_n = \frac{4n-3}{3n+4} = \frac{4-3/n}{3+4/n} \to \frac{4}{3} \qquad \text{as } n \to \infty.$$

18.

$$a_n = \frac{n^{1/3}+n^{1/4}}{n^{1/2}+n^{1/5}} = \frac{n^{-1/6}+n^{-1/4}}{1+n^{-3/10}} \to 0 \qquad \text{as } n \to \infty.$$

22. $a_n = \sin(n\pi/2)$ and so $a_n = 0$ for even n. For odd n, we note that $|a_n| = 1$ and so the sequence diverges.

32.

$$a_n = \ln(n+1) - \ln n = \ln\frac{n+1}{n} = \ln\Big(1+\frac{1}{n}\Big) \to 0 \qquad \text{as } n \to \infty.$$

38.

$$\begin{aligned} a_n &= (\sqrt{n+1}-\sqrt{n}\,)\sqrt{n+\frac{1}{2}} = \frac{\sqrt{n+1/2}}{\sqrt{n+1}+\sqrt{n}} \\ &= \frac{\sqrt{1+1/2n}}{\sqrt{1+1/n}+1} \to \frac{1}{2} \qquad \text{as } n \to \infty. \end{aligned}$$