EXAM 1 **Name:**
Math 2513
10/14/20

PROBLEM 1. *(10 points)* Here is a purported proof that the integer 2 is smaller than the integer 1. Find any incorrect conclusions that are made in the "proof" and carefully explain what makes them invalid.
Claimed Prooof: Let a and b be integers and assume that $a < b$. It follows that $a^2 < b^2$ and this inequality can then be rewritten as

$$0 < b^2 - a^2 = (b-a)(b+a).$$

Dividing both sides of this inequality by the positive number $b - a$ shows that $b + a > 0$. Subtracting a from both sides of the equation gives $-a < b$. Now imagine choosing $a = -2$ and $b = 1$. Since $-2 < 1$ the argument above shows that $-(-2) = -a < b = 1$. Therefore $2 < 1$. □

PROBLEM 2. *(30 points)* Give an element-wise proof of:

Proposition: For all sets A, B and C, $A \cap (B \cup C) \subseteq (A \cap B) \cup C$.

PROBLEM 3. *(20 points)* Let A, B and C be sets.
(a) Give a counterexample showing that $(A \cup B) - C$ need not be a subset of $(A - B) \cup C$.
(b) If A, B and C are sets where $(A \cup B) - C$ is a subset of $(A - B) \cup C$ what does that say about the relationship between the three sets?

PROBLEM 4. *(15 points)* In this problem B and C are sets. Use an element-wise approach to prove the proposition that "*If $B \subseteq C$ then $B - C = \emptyset$.*"

PROBLEM 5. *(10 points)* Consider the statement:

$$\forall m \in \mathbb{N}, \exists n \in \mathbb{Z} \text{ such that } m^2 + n^2 \text{ is odd}$$

(a) Write the statement as a concise English sentence.
(b) Write the negation of the statement as a concise English sentence.
(c) Verify that the statement is true.

PROBLEM 6. *(15 points)* We say that an sg-path in the integer grid has property G provided that there are no two successive U's in the RU-string associated with the path.
(a) List the RU-strings for all of the sg-paths from $(0, 0)$ to $(2, 2)$ that have property G.
(b) There are 120 sg-paths from $(0, 0)$ to $(3, 7)$. How many of them have property G? Explain.
(c) There are 120 sg-paths from $(0, 0)$ to $(7, 3)$. How many of them pass through the grid point $(2, 2)$ and have property G? Explain.
(d) How many of the sg-paths from $(0, 0)$ to $(7, 3)$ have property G?