Math 2924, Problem Set, November 12

1. Find the radius and interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{2n+3}{2^n(n-1)} x^n$.

2. Find the radius and interval of convergence for $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$.

3. The geometric series shows that $\dfrac{1}{1-x}$ has power series representation $\displaystyle\sum_{n=0}^{\infty} x^n$ for $-1 < x < 1$. Use this to find power series representations for each of the following. Also indicate the interval of convergence in each case.

   (a) $\dfrac{1}{1-(x/2)}$ (b) $\dfrac{1}{2-x}$ (c) $\dfrac{x^2}{1-x}$ (d) $\dfrac{1}{1+x}$ (e) $\dfrac{1}{1+7x}$ (f) $\dfrac{1}{1-x^2}$ (g) $\dfrac{x}{1-x^2}$

4. Part (e) of the previous problem might be solved by first determining the partial fraction decomposition for $x/(1-x^2)$, and then finding power series for each piece. Carry this out and show that your answer agrees with your answer from the previous problem.

5. Let $\displaystyle\sum_{n=1}^{\infty} c_n x^n$ be a power series whose radius of convergence is 3 and whose interval of convergence is $I = (-3,3]$. Determine the radius and interval of convergence for each of:

   (a) $\displaystyle\sum_{n=1}^{\infty} 3c_n x^n$ (b) $\displaystyle\sum_{n=5}^{\infty} c_n x^n$ (c) $\displaystyle\sum_{n=1}^{\infty} c_n x^{n+3}$ (d) $\displaystyle\sum_{n=1}^{\infty} c_n 5^n x^n$

   (e) $\displaystyle\sum_{n=1}^{\infty} c_n 5^n x^n / 3^n$ (f) $\displaystyle\sum_{n=1}^{\infty} (-1)^n c_n x^n$ (g) $\displaystyle\sum_{n=1}^{\infty} c_n x^{2n}$ (h) $\displaystyle\sum_{n=1}^{\infty} (-1)^n c_n x^{2n}$

6. (a) Determine the intervals of convergence for each of:

$$\sum_{n=0}^{\infty} \frac{x^n}{n}, \qquad \sum_{n=0}^{\infty} \frac{(-1)^n}{n} x^n, \qquad \sum_{n=0}^{\infty} \frac{x^{2n}}{n}, \qquad \sum_{n=0}^{\infty} \frac{x^n}{n^2}$$

. (b) Let a be a fixed positive number. Find four power series whose respective intervals of convergence are $(-a, a)$, $[-a, a)$, $(-a, a]$ and $[-a, a]$.

7. Let $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ be a power series whose radius of convergence is $7/2$.   (a) What is the radius of convergence for $\displaystyle\sum_{n=0}^{\infty} c_n (x - 7/2)^n$? What are the possible intervals of convergence for this series?

(b) What is the radius of convergence of $\displaystyle\sum_{n=0}^{\infty} \frac{c_n}{n} x^n$