Math 6813 homework

Problems in parentheses are optional— not necessary, but you might find it interesting to think about them. You are responsible for the rest of the listed problems, that is, you should be able to do them if asked (say, on a midterm exam). Some are exercises that are not worth writing out in detail if you understand how to do them. Only the starred problems need to be turned in.

1. (complete by 9/2) Review the appendix on CW-complexes in the Hatcher text.

2. (due 9/11) 0.1 (Hint: Think of the torus minus a point as the square minus an interior point, with the sides identified in the usual construction of the torus. The identified sides form the two circles. I don't think it is necessary to have explicit formulas for the deformation, just understand what is going on.), 0.2, 0.3 (for 0.3 and many of the other problems, one uses the fact that if $g \simeq g'$ then $fg \simeq fg'$ and $gh \simeq g'h$ for any f and h for which these make sense), 0.4*, (0.5), (0.6a), 0.9* (Hint: there is an extremely short proof of this), 0.10*, 0.11*, (0.12)

3. (9/11) 0.14*, 0.17*

4. (9/23) 0.18, 0.19, 0.20*, 0.21*, 0.23

5. (9/23) (0.16 This one is optional.) Hint: Start with the CW-structure that has two n-cells e^n_+ and e^n_- in each dimension (the upper and lower hemispheres). Use the HEP to show that if $F\colon S^\infty \to S^\infty$ is a map with $F(e^0_+) = e^0_+$, then there is a homotopy R with $R_0 = F$, $R_t(e^0_+) = e^0_+$, and $R_1(e^n_+) = e^0_+$. Telescope some of these together to produce a homotopy from 1_{S^∞} to $c_{e^0_+}$.)

6. (9/23) 0.26* (Hint: The first step is to show that the inclusion $i\colon X\times\{0\}\cup A\times I \to X\times I$ is a homotopy equivalence. The argument uses the obvious deformation retraction from $X \times \{0\} \cup A \times I$ to $X \times \{0\}$, and the inclusion $j\colon X \times \{0\} \to X \times I$. After applying Corollary 0.20, the proof of Proposition 0.18 can be adapted. You don't need to fill in all its details for Proposition 0.18, but fill in some if you want.)

7. (10/7) 1.1.1, 1.1.2, 1.1.3* (first check that $\beta_g\beta_h^{-1}$ is conjugation by $[g*\overline{h}]$), 1.1.5*, 1.1.6*

8. (10/7) 1.1.8, 1.1.10, 1.1.11, 1.1.12*, 1.1.13* (the last words in the problem should be "in A is path homotopic to a path in A.")

9. (10/23) 1.1.15, 1.1.17, 1.1.18*, 1.2.1*

10. (10/23) 1.2.2*, 1.2.3* (induct on the number, for A_1 take the interior of an n-ball centered at p and disjoint from the other points)

11. (10/23) 1.2.4 (one way is to show that X deformation retracts to a wedge of circles)

12. (10/23) Study example 1.25 of Section 1.2.