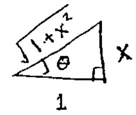
§ 6.6

Simplify the expression.

14) $\cos(2\tan^{-1} x)$ Let $\tan^{-1} x = \theta \longrightarrow \tan\theta = \frac{x}{1} \longrightarrow$ 

[$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$. The hypotenuse is a consequence of the Pythagorean Theorem].

$\cos(2\tan^{-1} x) = \cos(2\theta) = \cos^2\theta - \sin^2\theta$ [Double angle formula for cosine]

$$= \left(\frac{1}{\sqrt{1+x^2}}\right)^2 - \left(\frac{x}{\sqrt{1+x^2}}\right)^2 \quad \text{[by triangle]}$$

$$= \underline{\frac{1-x^2}{1+x^2}}$$

Find the derivative.

28) $F(\theta) = \arcsin\sqrt{\sin\theta}$ $\frac{d}{d\theta} F(\theta) = \frac{d}{d\theta}\left[\arcsin\sqrt{\sin\theta}\right] = \frac{1}{\sqrt{1+[\sqrt{\sin\theta}]^2}} \cdot \frac{d}{d\theta}\left[\sqrt{\sin\theta}\right]$

$$= \frac{1}{\sqrt{1+\sin\theta}} \cdot \frac{1}{2}[\sin\theta]^{-\frac{1}{2}} \frac{d}{d\theta}[\sin\theta] = \frac{1}{\sqrt{1+\sin\theta}} \cdot \frac{1}{2\sqrt{\sin\theta}} \cdot \cos\theta = \underline{\frac{\cos\theta}{2\sqrt{\sin\theta}\sqrt{1+\sin\theta}}}$$

Evaluate the integral.

62) $\int_0^{\sqrt{3}/4} \frac{dx}{1+16x^2} = \int_0^{\sqrt{3}/4} \frac{dx}{1+(4x)^2}$

Let $4x = \tan\theta$

$\rightarrow 4dx = \sec^2\theta \, d\theta \longrightarrow \frac{1}{4}\sec^2\theta \, d\theta = dx$

For bounds: $\theta = \tan^{-1}(4x)$

$x = 0 \Rightarrow \theta = 0$

$x = \sqrt{3}/4 \Rightarrow \theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$$= \frac{1}{4}\int_0^{\pi/3} \frac{\sec^2\theta \, d\theta}{1+\tan^2\theta} = \frac{1}{4}\int_0^{\pi/3} d\theta$$

$$= \frac{1}{4}\theta \Big|_0^{\pi/3} = \frac{1}{4}\left(\frac{\pi}{3}\right) = \underline{\frac{\pi}{12}}$$

§ 6.7

Find the derivative. Simplify where possible.

40) $y = \sinh^{-1}(\tan x)$

$$\frac{d}{dx}[y] = \frac{d}{dx}\left[\sinh^{-1}(\tan x)\right] = \frac{1}{\sqrt{1+[\tan x]^2}} \cdot \frac{d}{dx}[\tan x] = \frac{1}{\sqrt{1+\tan^2 x}} \cdot \sec^2 x$$

$$= \frac{\sec^2 x}{\sqrt{\sec^2 x}} = \underline{\sec x}$$

Evaluate the integral.

62) $\int \tanh x \, dx = \int \frac{\sinh x}{\cosh x} dx = \int \frac{1}{u} du = \ln|u| + C = \underline{\ln|\cosh x| + C}$

Let $u = \cosh x$

$\rightarrow du = \sinh x \, dx$