Homework # 13

7.4

6. (a) $\dfrac{t^6+1}{t^6+t^3} = \dfrac{(t^6+t^3)-t^3+1}{t^6+t^3} = 1 + \dfrac{-t^3+1}{t^3(t^3+1)} = 1 + \dfrac{-t^3+1}{t^3(t+1)(t^2-t+1)} = 1 + \dfrac{A}{t} + \dfrac{B}{t^2} + \dfrac{C}{t^3} + \dfrac{D}{t+1} + \dfrac{Ex+F}{t^2-t+1}$

(b) $\dfrac{x^5+1}{(x^2 - x)(x^4+2x^2+1)} = \dfrac{x^5+1}{x(x-1)(x^2+1)^2} = \dfrac{A}{x} + \dfrac{B}{x-1} + \dfrac{Cx+D}{x^2+1} + \dfrac{Ex+F}{(x^2+1)^2}$

8. $\displaystyle\int \frac{3t-2}{t+1}\,dt = \int \left(3 - \frac{5}{t+1}\right) dt = 3t - 5\ln|t+1| + C$

20. $\dfrac{x^2-5x+16}{(2x+1)(x-2)^2} = \dfrac{A}{2x+1} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2} \quad\Rightarrow\quad x^2-5x+16 = A(x-2)^2 + B(x-2)(2x+1) + C(2x+1).$

Setting $x = 2$ gives $10 = 5C$, so $C = 2$. Setting $x = -\frac{1}{2}$ gives $\frac{75}{4} = \frac{25}{4}A$, so $A = 3$. Equating coefficients of x^2, we get $1 = A + 2B$, so $-2 = 2B$ and $B = -1$. Thus,

$$\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx = \int \left(\frac{3}{2x+1} - \frac{1}{x-2} + \frac{2}{(x-2)^2}\right) dx = \frac{3}{2}\ln|2x+1| - \ln|x-2| - \frac{2}{x-2} + C$$

7.5

18. Let $u = \sqrt{t}$. Then $du = \dfrac{1}{2\sqrt{t}}\,dt \quad\Rightarrow\quad \displaystyle\int_1^4 \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt = \int_1^2 e^u\,(2\,du) = 2\left[e^u\right]_1^2 = 2(e^2 - e).$

46. Use integration by parts with $u = (x-1)e^x$, $dv = \dfrac{1}{x^2}\,dx \quad\Rightarrow\quad du = [(x-1)e^x + e^x]\,dx = xe^x\,dx$, $v = -\dfrac{1}{x}$. Then

$$\int \frac{(x-1)e^x}{x^2}\,dx = (x-1)e^x\left(-\frac{1}{x}\right) - \int -e^x\,dx = -e^x + \frac{e^x}{x} + e^x + C = \frac{e^x}{x} + C.$$