Homework 14 Solutions

7.8 #2. (a) proper, (b) Type 2, because it has discontinuity at $x = \pi/2$, (c) Type 2, because it has discontinuity at $x = -1$, (d) Type 1, because the interval of integration is infinite.

7.8 #10. The integral is improper because the interval of integration is infinite. We compute

$$\begin{aligned}\int_{-\infty}^{0} 2^r\,dr &= \lim_{p \longrightarrow -\infty} \int_{p}^{0} 2^r\,dr = \lim_{p \longrightarrow -\infty} \left.\frac{2^r}{\ln 2}\right|_p^0 \\ &= \lim_{p \longrightarrow -\infty} \left[\frac{1}{\ln 2} - \frac{2^p}{\ln 2}\right] = \frac{1}{\ln 2} - 0 = \frac{1}{\ln 2}.\end{aligned}$$

7.8 #24. The integral is improper because the interval of integration is infinite. We compute

$$\int_0^{\infty} \frac{e^x}{e^{2x}+3}dx = \lim_{p \longrightarrow \infty} \int_0^{p} \frac{e^x}{e^{2x}+3}dx.$$

But

$$\begin{aligned}\int \frac{e^x}{e^{2x}+3}dx &= \int \frac{e^x}{(e^x)^2 + (\sqrt{3})^2}dx \\ &= \frac{1}{\sqrt{3}} \arctan \frac{e^x}{\sqrt{3}} + C\end{aligned}$$

Therefore, we obtain

$$\begin{aligned}\int_0^{\infty} \frac{e^x}{e^{2x}+3}dx &= \lim_{p \longrightarrow \infty} \int_0^{p} \frac{e^x}{e^{2x}+3}dx \\ &= \lim_{p \longrightarrow \infty} \frac{1}{\sqrt{3}} \arctan \left.\frac{e^x}{\sqrt{3}}\right|_0^p \\ &= \frac{1}{\sqrt{3}} \lim_{p \longrightarrow \infty} \left[\arctan \frac{e^p}{\sqrt{3}} - \arctan \frac{1}{\sqrt{3}}\right] \\ &= \frac{1}{\sqrt{3}} \left(\frac{\pi}{2} - \frac{\pi}{6}\right) = \frac{\pi\sqrt{3}}{9}.\end{aligned}$$

7.8 #28. The integral is improper because it has infinite discontinuity at $x = 3$. We compute

$$\begin{aligned}\int_2^3 \frac{1}{\sqrt{3-x}}dx &= \lim_{p\longrightarrow 3^-} \int_2^p \frac{1}{\sqrt{3-x}}dx \\ &= \lim_{p\longrightarrow 3^-} -2(3-x)^{1/2}\Big|_2^p \\ &= -2\lim_{p\longrightarrow 3^-} [(3-p)-(3-2)] \\ &= 2.\end{aligned}$$

7.8 #36. The integral is improper because it has infinite discontinuty at $x = \pi$. We compute

$$\begin{aligned}\int_{\pi/2}^{\pi} \csc x \ dx &= \lim_{p\longrightarrow \pi^-} \int_{\pi/2}^{p} \csc x \ dx \\ &= \lim_{p\longrightarrow \pi^-} \ln|\csc x - \cot x|\Big|_{\pi/2}^{p} \\ &= \lim_{p\longrightarrow \pi^-} [\ln|\csc p - \cot p| - \ln(1-0)] \\ &= \infty, \quad \text{DIVERGENT.}\end{aligned}$$

THE END