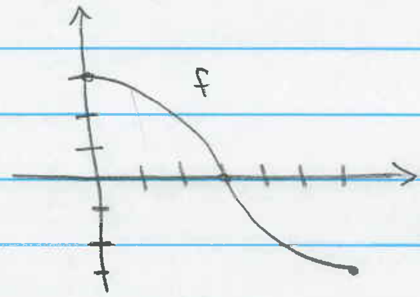
Calculus 2 Homework #3

4.3 4, 14, 20, 32, 4.4 32

(4) Let $g(x) = \int_0^x f(t)\,dt$, where f is shown on the graph.



(a) Evaluate g(0).

$g(0) = \int_0^0 f(t)\,dt = 0$

$g(6) = \int_0^6 f(t)\,dt$

$= \int_0^3 f(t)\,dt + \int_3^6 f(t)\,dt$

$= A_+ + A_-$

$= 0$

(b) Estimate g(x) for x = 1, 2, 3, 4, and 5

$g(1) = \int_0^1 f(t)\,dt \approx 2.7$

$g(2) = \int_0^2 f(t)\,dt \approx 4.8$

$g(3) = \int_0^3 f(t)\,dt \approx 5.5$

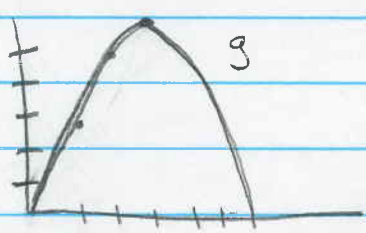
$g(4) = \int_0^4 f(t)\,dt \approx 4.8$

$g(5) = \int_0^5 f(t)\,dt \approx 2.7$

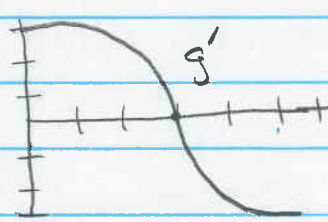
(c) g is increasing from 0 to 3.
Note the area under the curve is above the x-axis on this interval.

(d) g achieves its maximum value at x=3.

(e)



(f)



Find $\frac{dh}{dx}$

(14) $h(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz$

We must use FTC and the Chain Rule on this problem

$$\frac{d}{dx}\int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz = \frac{d}{dx}\int_1^{u} \frac{z^2}{z^4+1}\,dz$$

Let $u = \sqrt{x}$ so $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$$= \frac{d}{du}\left[\int_1^{u} \frac{z^2}{z^4+1}\,dz\right]\frac{du}{dx}$$

by chain rule

$$= \frac{u^2}{u^4+1}\,\frac{du}{dx}$$

by FTC

$$= \frac{x}{x^2+1}\cdot\frac{1}{2\sqrt{x}}$$

by replacing u w/ $\sqrt{x}$

$$= \frac{\sqrt{x}}{2(x^2+1)}$$

(20) $\int_{-1}^{1} x^{100}\,dx = \frac{1}{101}x^{101}\Big]_{-1}^{1}$

$$= \frac{1}{101}\left[(1)^{101} - (-1)^{101}\right]$$

$$= \frac{1}{101}\left[1 - (-1)\right]$$

$$= \frac{2}{101}$$

(32) $\int_0^{\pi/4} \sec\theta \tan\theta \, d\theta = \sec\theta \Big]_0^{\pi/4}$

$= \sec\frac{\pi}{4} - \sec 0$

$= \frac{1}{\cos \pi/4} - \frac{1}{\cos 0}$

$= \frac{1}{\sqrt{2}/2} - 1$

$= \frac{2}{\sqrt{2}} - 1$

4.4 (32) $\int_{\pi/4}^{\pi/3} \csc^2\theta \, d\theta = -\cot\theta \Big]_{\pi/4}^{\pi/3}$

$= -\cot \pi/3 - (-\cot \pi/4)$

$= \frac{-\cos \pi/3}{\sin \pi/3} + \frac{\cos \pi/4}{\sin \pi/3}$

$= \frac{-1/2}{\sqrt{3}/2} + \frac{\sqrt{2}/2}{\sqrt{2}/2}$

$= \frac{-1}{\sqrt{3}} + 1$