§ 4.4

50. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100+\int_0^{15} n'(t)\,dt$ represent?

$100+\int_0^{15} n'(t)\,dt$ represents the honeybee population after 15 weeks.

§ 4.5

Evaluate the integral.

10. $\int (3t+2)^{2.4}\,dt$

Let $u = 3t+2$. Then $du = 3\,dt \Rightarrow \frac{1}{3}du = dt$

So $\int (3t+2)^{2.4}\,dt = \int (u)^{2.4}\cdot\frac{1}{3}du = \frac{1}{3}\int u^{2.4}\,du = \frac{1}{3}\cdot\frac{u^{3.4}}{3.4}+C = \frac{5}{51}(3t+2)^{3.4}+C$

16. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Let $u=\sqrt{x}$. Then $du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2\,du = \frac{dx}{\sqrt{x}}$

So $\int \sin\sqrt{x}\cdot\frac{dx}{\sqrt{x}} = \int \sin(u)\cdot 2\,du = 2\int \sin(u)\,du = 2(-\cos(u))+C$

$= -2\cos(\sqrt{x})+C$

Evaluate the definite integral.

38. $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$

Let $u = x^2$. Then $du = 2x\,dx \Rightarrow \frac{1}{2}du = x\,dx$. When $x=0$, $u=(0)^2=0$.
When $x=\sqrt{\pi}$, $u=(\sqrt{\pi})^2=\pi$

So $\int_0^{\sqrt{\pi}} \cos(x^2)\cdot x\,dx = \int_0^{\pi}\cos(u)\cdot\frac{1}{2}du = \frac{1}{2}\int_0^{\pi}\cos(u)\,du = \frac{1}{2}\sin(u)\Big|_0^{\pi} = \frac{1}{2}[\sin(\pi)-\sin(0)]$

$= \frac{1}{2}[0-0] = 0$

48. $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$

Let $u = 1+2x$. Then $du = 2\,dx \Rightarrow \frac{1}{2}du = dx$. Also, $\frac{u-1}{2} = x$.

When $x=0$, $u = 1+2\cdot 0 = 1$. When $x=4$, $u = 1+2\cdot 4 = 9$.

So $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx = \int_1^9 \frac{\left(\frac{u-1}{2}\right)}{\sqrt{u}}\cdot\frac{1}{2}du = \frac{1}{4}\int_1^9 \frac{u-1}{\sqrt{u}}\,du = \frac{1}{4}\int_1^9 (u^{1/2}-u^{-1/2})\,du$

$= \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]\Big|_1^9 = \frac{1}{4}\left(\left[\frac{2}{3}(9)^{3/2}-2(9)^{1/2}\right]-\left[\frac{2}{3}(1)^{3/2}-2(1)^{1/2}\right]\right)$

$= \frac{1}{4}\left([18-6]-[\frac{2}{3}-2]\right) = \frac{1}{4}\left[12+\frac{4}{3}\right] = 3+\frac{1}{3} = \frac{10}{3}$